ADDICTION TO REWARDS*

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Abstract

The paper explores the "hidden costs of rewards" in a dynamic principal-agent framework, in which an informed principal selects in each period a reward for the agent. It shows that rewards are addictive in that once offered, a contingent reward makes an agent expect it whenever a similar task is faced, which, in turn, compels a principal to use rewards over and over again. Furthermore, in a long-term principal-agent relationship there is a double-sided ratchet effect: the principal is concerned about creating addiction for the agent, whereas the agent wants to conceal his self-confidence. On the principal's side, the ratchet effect implies that there are fewer rewards in a long-term principal-agent relationship than in a situation where the agent faces transient principals while implementing a series of similar tasks. On the agent's side, ratcheting conflicts with a desire to work in order to acquire the relevant information.

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1 Introduction

Traditional economic analysis studies the effects of extrinsic incentives on human behavior. Contingent incentive schemes, from the simplest piece rates for industrial workers to sophisticated compensation packages of top executives, are not only recommended by the theory, but are also widely and, often, successfully[1] used in practice. Yet, since the early seventies many psychologists became concerned with potential "hidden costs of rewards". In a pioneering work, Deci (1971) argued that some activities are intrinsically motivated – provide their own inherent reward – and application of explicit external rewards can be harmful for the intrinsic motivation.

Starting with Deci's work, a large number of experiments have been conducted investigating the impact of different types of external rewards (e.g. uncontingent versus contingent, monetary versus verbal etc.) on intrinsic motivation. Deci and Ryan (1985) in their book on intrinsic motivation, and, more recently, a meta-analysis of empirical work by Deci, Koestner and Ryan (1999) report extensive evidence of detrimental effects of rewards, collected during the last three decades. Yet, some divergence in the empirical findings,[2] their strong sensitivity to the design of experiments suggest compellingly that particular conditions need to be satisfied for the rewards to impair intrinsic motivation. Citing Lepper et al. (1999), "...the relevant issue for further research was not whether rewards have negative, or positive effects "in general" but rather when and why these different effects might occur".

Economists also have been concerned with the dangers of performance-dependent rewards. Adam Smith (cited in Laffont-Martimort (2001)) wrote that "workmen... when they are liberally paid by the piece, are very apt to overwork themselves, and to ruin their health and constitution in a few years". There is a large recent experimental literature exploring the "crowding out" of intrinsic motivation in different contexts.[3]

What are the undesired effects of rewards? To begin with, a small (inadequate) reward may cause an immediate reduction in effort. A paper by Gneezy and Rustichini (2000) with an eloquent title "Pay enough or don't pay at all" describes experiments in which an introduction of a miserable piece-rate component significantly decreased the students' performance on an IQ-type test relative to an unconditional flat payment, whereas moderate bonuses increased performance.[4]

There are also important long-term costs. If a reward is offered for the completion

[1]See, for example, Lazear (2000), who reports a significant increase of workers' productivity after the introduction of a piece-rate scheme on a large manufacturing company.

[2]See, for example, Cameron and Pierce (1994) and Eisenberger, Pierce and Cameron (1999) for a more sceptical view on the the negative effects of rewards.

[3]For example, Frey and Oberholzer-Gee (1997), Frey et al. (2000), Frey (1997), Falk et al. (1999).

[4]A theoretical paper suggesting a possible explanation for such discontinuity in behavior in some situations is Seabright (2002): "virtuous" people may value the ability to signal their type by performing certain actions for free, and a small payment may deprive them of this ability by pooling with less "virtuous" counterparts.

of a task which was interesting in itself, the subject will still choose to fulfill the job if the reward is high enough, but the attitude to the task may be spoiled, as measured both by "self-reports" and the future willingness to re-engage in similar activity. In other words, being a (weakly) positive short-term reinforcer, a reward may become a negative reinforcer in the long-run. A recent paper by Bénabou and Tirole (2001) (B&T in what follows) suggest a possible mechanism through which motivation may be hurt by rewards in this way. In their "intrinsic motivation" model an agent performs a task which brings benefits both to him and to a principal. The agent (he) is uncertain about the potential gains from the project and, depending on his anticipation, would choose to work diligently or lazily on a stand-alone basis. Then, if the principal (she) is better informed than the agent about the general attractiveness of the task (be it the agent's ability to perform the task, his ultimate payoff, or the true cost of effort), her remuneration policy or any other move (help, coaching, delegation, etc.) will convey some of this relevant information. As B&T show, a promise of high performance-contingent reward bears a pessimistic message about the agent's ability and injures his self-confidence. The impact is two-fold: In the short run, the agent's overall incentive weakly increases. Conversely, in the long run, the reward impairs self-confidence.

However, the costs of rewards that B&T consider are of a dynamic nature, whereas their model considers only a punctual relationship between the principal and the agent. In particular, the principal cares only about the short-term effect of self-confidence impairment neglecting the long-term consequences. Similarly, the agent does not try to avoid appearing too enthusiastic about the task if the relationship is not repeated or is not observed by the future principals.

This paper applies B&T's approach to a repeated interaction. I model a two-period informed-principal setting and compare two different situations: in one, the agent (he) interacts with the same principal (she) in both periods; in the other, he faces a new principal in the second period. I initially assume that it is the agent's ability (or suitability for a particular task) which he is uncertain about (later an alternative formulation with the agent's payoff unknown is considered). Then, unless the agent's type was revealed by the principal's policy in the first period, some uncertainty remains in the second period. In this case, both with a permanent and transitory principals, a pattern where higher bonuses are promised to weaker agents is observed in each period.

Bad news brought by a high first-period bonus makes the agent addicted to rewards. Thus, if the principal offers a high bonus in the first period, she (or a subsequent alternative principal) is doomed to give it again in the second. Unlike the agent, who gets addicted involuntarily, without choosing whether to be rewarded, the principal (in the model with a permanent principal) experiences a sort of "rational addiction" in the Beckerian sense.[5] In the model with transient principals, the second principal inherits the need to give rewards from her first colleague.

[5] Stigler and Becker (1977), Becker and Murphy (1988).

When the agent interacts with a long-term principal, there is a double-sided ratchet effect:[6] the agent attempts to signal his low self-confidence through a low effort in the first period (hoping to be offered a bonus in the second), whereas the principal is concerned about offering a bonus in the first period as in this case she will have to give it again in the second period. With transitory principals, the second ratchet effect disappears, and bonuses are given more lavishly in the first period. This can be called a "grand-parent" effect. Indeed, when grand-parents occasionally look after the child, they often tend to be much more permissive with him than the parents. One of the reasons is that the grand-parents do not care too much about the long-term consequences of their indulgence as long as they do not see the child too often.

Another strategic aspect which is absent in the punctual relationship is that of the information acquisition. It is natural to assume that by choosing to be active – work hard – the agent obtains some relevant information[7]. For example, when the unknown parameter is the agent's ability, i.e. his chance to succeed, the outcome of the first project – success or failure – will be informative about ability. When the cost of effort is unknown, exerting effort should lead to observation (at least noisy) of this cost. Thus, in the early stage of the relationship an agent with an intermediate level of self-confidence (or perception of the task) faces a trade-off between shirking in order to solicit bonus in the second period and working in the hope to reduce the informational disadvantage with respect to the principal. This conflict between passivity/concealing one's private information and activity/obtaining new information may be observed in other circumstances (e.g. asking questions about others' research or bargaining).

If the evidence the agent obtains at the interim stage of relationship allows complete elimination of the agent's uncertainty, and exerting effort in the first period is instrumental in obtaining the evidence, the principal may become more prone to stimulate effort, and thus learning, when the information is favorable. Then, bonuses no longer undermine the agent's self-confidence and addiction does not develop.

The paper is organized as follows: Section 2 introduces a version of "intrinsic motivation" model of B&T. Section 3 analyzes a two-period version of the model in which the agent faces different principals. Section 4 studies the two-period model with one long-term principal and compares it with the model with two principals. Section 5 considers alternative formulations of the model. Section 6 concludes. All proofs and some technical results are relegated to Appendix.

[6] See, for example, Laffont and Tirole (1988) for a general analysis of the ratchet effect.

[7] This effect can be termed "learning by doing". However, usually learning by doing means improving one's skills by practice, whereas here it means just finding out one's skills (or preferences).

2 The Static Model

This section describes a simple two-type version of the intrinsic motivation model introduced in B&T and recaps some of their results that will be useful for further exposition. I shall present the model first, then discuss its possible interpretations and applications and finish by describing its equilibria.

2.1 Description of the Model

The model describes a principal-agent relationship. The agent (he) works on a project, which, in case of success, brings him a benefit $V > 0$. The principal (she) has a stake in the agent's project: she gets $W > 0$ if the project succeeds.

The agent's ability and his unobservable effort e are both essential for success. Ability parameter θ can assume two values θ_H and $\theta_L < \theta_H$ in $(0, 1)$. The agent has high ability with probability f_H and low ability with probability $f_L = 1 - f_H$. Effort e can be either low, $e = 0$, or high, $e = 1$; high effort costs c to the agent, low effort costs nothing. The probability of success is θe. The outcome is denoted by y , so that $y = 1$ if the agent succeeds in the task and $y = 0$ if he fails.

Both the principal and the agent are assumed to be risk-neutral, the agent's outside option is zero. To boost the agent's motivation to work hard, the principal may offer a contingent compensation scheme: a fixed wage a and a bonus payment b in case of success. Thus, the agent's expected payoff is

$$U^A = \theta e(V + b) + a - ce$$

and the principal's expected payoff is

$$U^P = \theta e(W - b) - a.$$

The agent is a supposed to have limited liability: the assumption is fairly standard and applicable to many situations. Initially, I also assume that the principal does not pay any fixed wage to the agent (i.e. $a \equiv 0$). I discuss the issue and introduce the possibility of paying lump-sum transfers in section 5.4. So far, since the only observable is the outcome y – success or failure – the only incentive scheme available is to pay a bonus b, assumed to be non-negative, in case of success and nothing in case of failure.[8] The agent will always accept any non-negative bonus offer and stay with the principal since he can secure his reservation utility by shirking. Moreover, the agent will get some positive ex ante rent in equilibrium.

[8] It is clearly optimal not to pay a bonus in case of failure. The restriction that bonuses be non-negative is justified if V is a non-monetary private benefit (the agent's limited liability is still at work) or if the agent can easily sabotage his observable performance without destroying V; the restriction can be easily relaxed.

Assumption 1 *Under full information, the low-type agent does not exert effort unless a positive bonus is promised; the high-type agent has sufficient intrinsic motivation to exert effort:*

$$\theta_L V < c < \theta_H V.$$

Notation 1 *The minimal bonus, required to induce effort of the low-type agent under full information is*

$$b^* = \frac{c}{\theta_L} - V.$$

Assumption 2 *Under full information the principal's payoff is high enough, so that she wants to induce effort of the low-type agent (i.e. in the first-best each type of agent should exert effort):*

$$W - b^* = W + V - \frac{c}{\theta_L} > 0.$$

I further assume that the agent has imperfect self-knowledge: he does not know θ, but observes an informative signal σ. In contrast, the principal knows θ, but does not observe σ and hence is unaware of the agent's estimate of his chances to succeed. Signal $\sigma \in \mathbb{R}_+$ the agent of type $i \in \{\theta_H, \theta_L\}$ receives has the cdf $G_i(\sigma)$ and the density function $g_i(\sigma)$.

Assumption 3 *Monotone Likelihood Ratio Property (MLRP):*
The likelohood ratio,

$$l(\sigma) := \frac{g_H(\sigma)}{g_L(\sigma)},$$

is increasing. Furthermore, $l(\sigma)$ has full support $[0, \infty)$.

Assumption 3 implies that high σ brings good news about the agent's type: The agent's estimate of his ability

$$\hat{\theta}(\sigma) = E[\theta|\sigma] = \frac{g_H(\sigma) f_H}{g_H(\sigma) f_H + g_L(\sigma) f_L}\theta_H + \frac{g_L(\sigma) f_L}{g_H(\sigma) f_H + g_L(\sigma) f_L}\theta_L \tag{1}$$

is an increasing function of σ.[9] Then, for each bonus b, offered by the principal, there is a threshold level $\sigma^*(b)$ such that the agent will exert effort if and only if he has received a signal above the threshold[10].

[9] Due to the MLRP,

$$\frac{d\hat{\theta}(\sigma)}{d\sigma} = \frac{d}{d\sigma}\left(\frac{l(\sigma) f_H \theta_H + f_L \theta_L}{l(\sigma) f_H + f_L}\right) = \frac{f_L f_H (\theta_H - \theta_L) l'(\sigma)}{(l(\sigma) f_H + f_L)^2} > 0.$$

[10] In particular, $\sigma^*(b)$ may be 0, in which case the agent works whatever signal he has received, or $\sigma^*(b)$ may be infinity so that the agent never works.

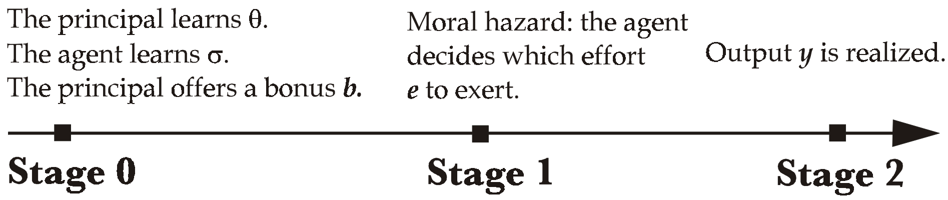

Figure 1: Timing in the one-period model.

Figure 1, presenting the timing, concludes the description of the model. As shown in the figure, I assume that the principal offers an explicit bonus at the initial stage of the game. More generally, according to the *inscrutability principle* (Myerson (1983)), the best thing the principal could do would be to offer a mechanism to which both the agent and the principal would report truthfully their private information (σ and θ respectively), and which would subsequently generate an incentive compatible reward scheme for the agent. However, since the only instrument available is the size of the bonus $b \geq 0$ (it is always optimal to pay nothing after failure), the principal cannot design a scheme which would depend non-trivially on the agent's signal σ and there is no loss of generality in considering explicit bonus offers.

2.2 Possible Interpretations and Applications

We can now discuss the assumptions and possible interpretations of the model. Since the paper is mostly motivated by the psychology literature the main interpretation of the model is that it describes an interpersonal interaction[11], such as a parent/child, teacher/student or manager/employee one. Often in such a relationship an activity that may be performed more or less eagerly by one of the parties (the agent) imposes a positive externality on the other party (the principal). For example, the parents may enjoy the fame of their child if he becomes a famous musician and thus push the telented kid to practice more without much caring about foregone enjoyments; similarly the parents are often concerned about lifelong effects of the child's education. A manager's salary or the probability of promotion is often sensitive to the quality of the work of her subordinates.[12]

As described in Section 2.1, the agent's *intrinsic motivation* may be high or low. According to Deci (1971), cited by Frey (1997), "one is said to be intrinsically

[11]The principal and the agent could also be, for example, two divisions of a firm: say, the research division imposes an externality on the sales division.

[12]This leaves open the question why the grand contract which specifies the manager's and the subordinate's functions within the firm is incomplete in that it does not describe the way they should interact. Without going into details, let us just admit that for some reason or another contracts often do leave some discretion to the managers in choosing incentives for the employees in particular tasks.

motivated to perform an activity when one receives no apparent reward except the activity itself". However, as Kreps (1997) notes, "what is called intrinsic motivation may be (at least in part) the worker's response to fuzzy extrinsic motivators, such as fear of discharge, censure by fellow employees, or even the desire for coworkers' esteem"[13]. Under a broader interpretation, accepting that more or less fuzzy extrinsic motivators are reflected in parameters c and V, the model seems applicable to many situations.[14] Moreover, sometimes it may be more natural to assume that the uncertainty is about the cost c or the value of success for the agent, V. An alternative formulation with uncertain V is considered in Section 5.

The probability of success, θ, is meant to reflect some combination of the agent's ability and fitness to the particular task (for brevity I will usually call it ability). Assuming that the principal knows it better than the agent, or at least that she has some relevant information that the agent does not have access to, seems natural in many circumstances: for example, in school it may be more difficult for a kid to form a judgment about his analytical abilities than for his teacher; the pupil may also fail to realize whether the problem he is tackling is similar to the ones he solved before or is conceptually different; the employer may be in a better position to assess the young employee's chances to do well on a new project. An important source of inaccuracy in the agent's self-image may also be selective memory or strategic neglect of some relevant information.[15]

I consider monetary bonuses, so giving b to the agent costs exactly b to the principal. In a number of applications, though, bonuses are non-monetary. For example, in a professor-student relationship a bonus might be a higher professor's involvement in the ensuing stages of the project; in a manager-employee relationship a bonus might be a pay raise or an attractive business trip arranged by the manager. Results of the paper extend to non-monetary rewards provided that paying a bonus is sufficiently costly for the principal (but not too costly - see Assumption 2).

2.3 Equilibria

An appropriate solution concept for the model is that of a Perfect Bayesian Equilibrium (PBE).[16] However, as we shall see in a moment, there are too many PBE in the game, and some restrictions on out-of-equilibrium beliefs will be imposed.

Analyzing a model with a continuum of types, B&T prove that a larger bonus, although it increases the probability that the agent will work hard, is offered in

[13]Seabright (2002) shows that another "fuzzy extrinsic motivator" to perform some seemingly unrewarding activity may be the desire to be matched with similar "altruistic" partners in future interactions.

[14]Assumptions $V, W > 0$ and $c > 0$ can be easily relaxed: see B&T.

[15]See Bénabou and Tirole (2002).

[16]See, for example, Fudenberg and Tirole (1991).

equilibrium only to a weaker type and thus brings bad news to the agent.[17] In the two-type model I consider they show that in any equilibrium the principal gives bonus $b \in [0, b^*]$ to the high-type agent and either randomizes between b and b^* or always gives b to the low-type agent. Moreover, it is easy to prove that, conversely, for any $b \in [0, b^*]$ there exists either a semi-separating equilibrium (the principal offers b to the high type and randomizes between b and b^* for the low type) or an equilibrium with complete pooling (the principal offers b regardless of the agent's type).

In order to analyze the dynamic model, we need to predict which equilibria will be played in continuation subgames. A version of Never a Weak Best Response (NWBR) criterion defined for signalling games in Cho and Kreps (1987) allows to select a unique equilibrium in the static game, and, as we shall see later, in each continuation subgame. In this monotonic signalling game this refinement is equivalent to "universal divinity" of Banks and Sobel (see, for example, Fudenberg and Tirole (1991)).

Assumption 4 *Out-of-equilibrium beliefs satisfy the NWBR criterion for signalling games.*

NWBR refinement imposes the following restriction: Assume that the set of the agent's best-response reactions[18] to some out-of-equilibrium bonus $\hat{b}$ that make the principal indifferent to deviating to $\hat{b}$ from some equilibrium bonus when the agent has type θ_i is strictly included in the set of best-response reactions that make the principal better off when the agent's type is θ_i'. Then, the agent should believe that his type is θ_i' when offered an out-of equilibrium bonus $\hat{b}$. In other words, if $\sigma_H^*(\hat{b})$ and $\sigma_L^*(\hat{b})$ describe the agent's reactions which make the principal indifferent between deviating to $\hat{b}$ or not when θ is equal to θ_H and θ_L respectively, the agent should believe that $\theta = \theta_H$ after receiving $\hat{b}$ if $\sigma_L^*(\hat{b}) < \sigma_H^*(\hat{b})$ and he should believe that $\theta = \theta_L$ if $\sigma_L^*(\hat{b}) > \sigma_H^*(\hat{b})$.

I now introduce some notation which will be used throughout the paper:

Notation 2

$$A := \left(\frac{f_L}{f_H}\right)\left(\frac{c - \theta_L V}{\theta_H V - c}\right),$$

$$\alpha := \frac{b^*}{W}.$$

[17]The intuition is the following: the principal who knows that the agent has the high type, also knows that the agent has a higher self-confidence on average, and thus she may rely less on extrinsic rewards (the *trust effect*). Besides, offering a given bonus to the high-type agent is more costly because $\theta_H > \theta_L$ implies that this bonus will be actually paid more frequently (the *profitability effect*).

[18]A best-response reaction for the agent in this game is (up to a modification on the set of signals of measure zero) to work whenever his signal exceeds some threshold σ^*.

Parameter A determines which effort the agent would choose in a hypothetical situation in which he obtained no information about θ – neither σ nor any bonus offer from the principal. The agent would work if $A \leq 1$ (i.e. when the expected loss from working when $\theta = \theta_L$ is smaller than the expected gain from working when $\theta = \theta_H$) and shirk otherwise[19]. Parameter α measures the size of the bonus b^* relative to the principal's payoff W.

Lemma 1 *(Bénabou-Tirole)Under Assumptions 1–4, there is a unique equilibrium. There exists a threshold value* $\tilde{f}_H$ *such that*[20]

i)if $f_H < \tilde{f}_H$, *the equilibrium is semi-separating: the principal offers no bonus to the high type and randomizes between no bonus (with probability* x^**) and bonus* b^* *(with probability* $1 - x^*$*) for the low type. The equilibrium strategies are determined by two equations:*

$$l(\sigma^*) = x^* A, \tag{2}$$

$$G_L(\sigma^*) = \alpha; \tag{3}$$

The pooling parameter x^* *increases with* f_H.

ii)if $f_H > \tilde{f}_H$, *the equilibrium is pooling: the principal never offers a bonus and the agent works if* σ *exceeds* σ^* *defined by*

$$l(\sigma^*) = A.$$

I refer the reader to B&T's Proposition 3 for a complete proof of Lemma 1. Besides, this result follows from a series of Lemmas in Section 5.4 concerning the model with lump-sum transfers. Just note, since similar equations will be frequently encountered in further analysis, that (3), which determines the agent's reaction for the case of semi-separation, expresses the principal's indifference between offering bonus b^* (in which case the agent works for sure) and offering no bonus (implying that the agent works with probability $1 - G_L(\sigma^*)$). Given σ^*, equation (2) expressing the agent's indifference between working or not when his signal is at the threshold level σ^*, determines the pooling parameter x^*. When the ex ante probability that $\theta = \theta_H$ is high, the principal never offers a bonus.

[19] The agent would work if $(f_H \theta_H + f_L \theta_L)V \geq c$, which is equivalent to $A \leq 1$.

[20] To see how $\tilde{f}_H$ is determined, note that A is a decreasing function of f_H, with $A \to \infty$ as $f_H \to 0$ and $A \to 0$ as $f_H \to 1$. Hence there exists $\tilde{f}_H$ such that $A = l(\sigma^*)$, where σ^* is determined implicitly by $G_L(\sigma^*) = \alpha$.

3 The Two-Period Model With Transitory Principals

3.1 Description

The model now has two periods. In each period the agent interacts with a different principal in a way described in the previous section. The two consecutive projects have identical characteristics: the agent's chances to succeed θ, the disutility of effort c, as well as the agent's and the principals' valuations V and W are the same in both periods. For notational simplicity, there is no discounting across periods.

At the beginning of the first period the first principal learns the agent's type and the agent learns signal σ. Then, the principal announces her bonus and the agent chooses his first-period effort; next, the first-period outcome is realized.

An important modelling choice is that of the information structure in the second period. I assume the agent does not receive new exogenous signals. However, if he exerted effort in the first period, he observes the outcome of the first project and thus updates the estimate of his ability: obviously, success brings good news about ability and failure brings bad news. Furthermore, unless the first principal's policy was completely uninformative (the pooling case), the agent will use in the second period the inference concerning θ he made from this policy choice.

The second principal observes the agent's type, the first principal's policy, the outcome of the first period and the agent's effort (whether he really tried) in the first period; like the first principal, she does not observe the agent's signal σ. Within the second period, events are the same as in the one-period model – the second principal announces her policy, the agent chooses whether to work or not and then the outcome is realized. Assuming that the second principal observes everything that happened in the first period makes analysis more transparent. Moreover, as will be shown later, the assumption that the first-period effort is observable is not necessary for obtaining the main results.[21]

3.2 Preliminary Results: the Structure of Equilibria

Any subgame which is played in the second period fits almost perfectly the description of the static model. There are some nuances, however. The first is that now the priors about the agent's type are different from those which the agent had before period one. For example, after being offered a first period bonus b_1 which the agent believes was given with probability $x_1^L > 0$ to the low-type agent and with probability $x_1^H > 0$ the high-type one, exerting effort in the first period ($e_1 = 1$) and

[21] Probably, the results are also robust to assuming that the second principal does not observe the first-period policy and outcome, but since I want to compare the present model with the one where there is just one principal, who naturally observes her own policy and payoff, exploring the model modified in this way is not very interesting.

suffering a failure ($y_1 = 0$), the agent will believe (before accounting for his private signal) that he has the high type with probability

$$\hat{f}_H = \frac{f_H x_1^H (1-\theta_H)}{f_H x_1^H (1-\theta_H) + f_L x_1^L (1-\theta_L)}. \tag{4}$$

Note that on the equilibrium path $\hat{f}_H$ is common knowledge, but this is not be true if the first principal offers an out-of-equilibrium bonus b_1 which may be interpreted differently by the agent and by the second principal. When the bonus is separating, (i.e. $x_1^L = 0$ or $x_1^H = 0$), the game virtually becomes a game of complete information. However, we need to specify what beliefs the agent will have if he receives two contradictory signals from two principals: say, the first-period bonus he receives is given in equilibrium to the low-type agent only and the second-period one is given to the high-type one. We make a natural assumption:[22]

Assumption 5 *If type-revealing bonus was offered in the first period, the agent sticks to beliefs induced by this bonus regardless of the second principal's policy.*

Another subtle difference with the static model of Section 2 is that now, since the second principal observes the agent's first-period effort, she knows that σ belongs to some subset $\Sigma_1 \subset \mathbb{R}_+$ if the agent exerted effort and to $\Sigma_0 = \mathbb{R}_+ \setminus \Sigma_1$ if he did not. Since the distribution functions for the signal conditional on these subsets, $G_i(\sigma|\Sigma_j)$, still satisfy MLRP[23], the only violation is of the full support assumption: the conditional likelihood ratio will in general assume values in a proper subset of $[0, \infty)$. The fact that very good signals may be excluded (i.e. the conditional likelihood ratio has a finite upper limit) does not change anything. On the other hand, when there are no really bad signals (i.e. the conditional likelihood has a large enough lower limit), the agent always works in the pooling equilibrium if $\hat{f}_H$ defined in (4) is large enough. Hence we obtain the following preliminary result:

Remark 1 *Each second-period subgame meets Assumptions 1 through 3 (save the full support assumption for the likelihood ratio) imposed on the static model. It has a unique (Perfect Bayesian) equilibrium satisfying Assumptions 4 and 5 which has the same structure as the one described in Lemma 1, but parameters x^*, σ^* are determined in a different way (specified later).*

[22] Although Assumption 5 seems natural, it is by no means the only possible one. Beaudry (1994) considers a similar two-period informed-principal model and assumes that the agent "forgets" his first-period beliefs in the second period. Then, Beaudry shows that twice repeated static outcome is a PBE of the two-period model and satisfies a generalized version of the intuitive criterion.

[23] $G_i(\sigma|\Sigma_j)$ is defined for $\sigma \in \Sigma_j$ and is equal to $\frac{G_i(\sigma|\Sigma_j)}{G_i(\Sigma_j)}$, so $l_j(\sigma) = \frac{g_H(\sigma|\Sigma_j)}{g_L(\sigma|\Sigma_j)} = l(\sigma)\frac{G_L(\Sigma_j)}{G_H(\Sigma_j)}$ is an increasing function.

In the following analysis I shall suppose that Assumptions 1–3 on the parameters hold, Assumption 4 is applied in both period to restrict the agent's out-of-equilibrium beliefs[24], and Assumption 5 constrains his second-period beliefs.

We can now analyze the agent's reaction to the first-period policy announcement. Note that it will be different from his reaction in the one-period model: the agent now has to take into account the impact of his choice of effort on the second principal's policy and on the information acquisition. Yet, as the following Lemma shows, the agent's behavior is similar to what he does in the static model.

Lemma 2 *In response to any bonus b_1 offered in the first period the agent exerts effort in the first period if and only if his signal is above some threshold $\sigma_1^*(b_1)$ (possibly, zero or infinite).*

Although the agents overall benefit from success in the first period is different from V, from Remark 1 it follows that it still would be optimal to work for the high-type agent and to shirk for the low-type one had the agent known his type[25], so Assumption 1 holds for the agent's first-period expected payoffs. Then, from Lemma 2 and the fact that the first principal's objective function is identical to that of the principal in the static model, the following preliminary result follows:

Remark 2 *The structure of equilibrium in the first period is as described in Lemma 1 (with parameter x_1^* determined in a different way). The agent's first period strategy in response to no bonus offer in the case of semi-separation is still defined by*

$$G_L(\sigma_1^*) = \alpha. \tag{5}$$

Now that we know the structure of the equilibrium of the whole game (existence and uniqueness under Assumptions 4 and 5 will be proved in the next subsection under some restrictions on parameters), it remains to determine parameters characterizing the players' strategies. For brevity, I shall only focus on the case that seems to be more interesting – that of semi-separating equilibrium.

3.3 Semi-Separating Equilibrium

In the second period the principal chooses a bonus b_2 as a function of the information available: $b_2 = b_2(\theta, b_1, e_1, y_1)$, where b_1, e_1 and y_1 stand for the first-period bonus, effort and outcome.

There are four possible types of continuation subgames: one happening after $b_1 = b^*$ and three other after $b_1 = 0$.

[24] NWBR refinement is designed for signalling games, but our model describes virtually two signalling games (with the second game depending on the outcome of the first), and, since the first principal does not care about the second period, NWBR can be applied to each of these games.

[25] The only motive to shirk in the first period for the able agent could be the hope to receive a positive bonus offer in the second period, but such hopes are vain: the second principal never offers a bonus to the high-type agent.

Notation 3 *When $b_1 = 0, e_1 = i$ and $y_1 = j$, the probability that the principal offers 0 to the low-type agent in the corresponding second-period subgame is denoted by x^*_{ij} and the signal threshold level, characterizing the agent's reaction in this subgame when no bonus is offered, is denoted by σ^*_{ij} .*

3.3.1 Case A: Agent revealed in the first period to be weak ($b_1 = b^*$).

If bonus b^* was given in the first period, the asymmetry of information about the agent's type disappears. After the first period the agent believes he has low ability (according to Assumption 5). Then, after a promise of b^* by the first principal, the second principal has no choice but to offer $b_2 = b^*$ in the second period. A trivial corollary follows:

Corollary 3 *Bonuses are addictive: once given, they go on: $b_2 = b^*$ whenever $b_1 = b^*$.*

The agent always works when offered bonus b^*.

3.3.2 Case B: No bonus offered in the first period ($b_1 = 0$).

The formal derivation of equilibrium strategies (that is σ^*_{ij}, x^*_{ij}) in subgames happening after no bonus offer in the first period is given in a series of Lemmas in Appendix A. Here I shall briefly and informally discuss the results.

Case B1: Success in the first period. In this case the agent received two good news: he had been offered no bonus in the first period and then tried and succeeded in the first task. These good news make bonuses redundant in the second period: the principal never offers a bonus in this subgame and yet the agent always works.

Case B2: Failure in the first period. A failure discourages the agent. However, after a failure the agent's self-confidence may still be high enough to encourage him to exert effort in the second period even if he was just willing to do so in the first ($\sigma = \sigma^*_1$): this happens if the ratchet effect – the agent's strategic shirking in the first period in the hope to get a bonus in the second period – is very strong. However, I shall confine the analysis to the case of semi-separation in this subgame, which, in case of semi-separation in the first period, implies that $\sigma^*_{10} > \sigma^*_1$, so the agent shirks after a failure if he is not offered a bonus. A necessary and sufficient condition for semi-separation in this subgame is that θ_H be large enough: then, failure brings really bad news about the agent's type (in the limit, when $\theta_H \to 1$, the agent learns that $\theta = \theta_L$ if a failure happens).

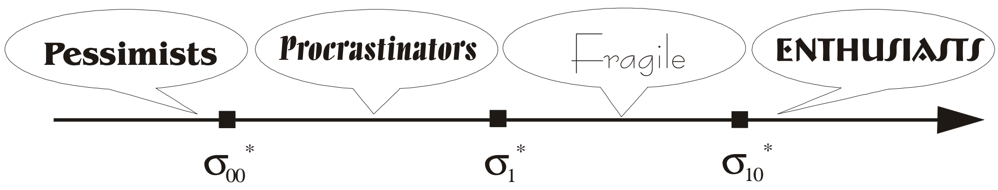

Figure 2: Four categories of agents.

Case B3: Agent did not try in the first period. In this subgame the equilibrium is necessarily semi-separating: If no bonus were ever offered in this subgame, no ratchet effect would be present in the choice of the first-period effort. Then, the agent would never work in this subgame because working in the first period is a better strategy than working only in the second since on top of direct benefits it leads to the acquisition of valuable information. This could not happen in equilibrium since the principal would rather pay b^* than let the agent always shirk in a subgame.

We now come back to the first period. When he chooses his first effort, the agent has two additional concerns on top of the usual comparison of immediate benefits versus losses. On the one hand, he wants to work more in the first period than he would do in the static model (for given beliefs about his type) because high effort, unlike shirking, leads to the acquisition of a valuable signal about his type that allows to make a better decision on whether to work in the second period. On the other hand, choosing not to work the agent may hope to obtain a bonus in the second period if he is the low-ability one – the ratchet effect.

With semi-separation in both periods when it is possible (i.e. $x_1, x_{00}, x_{10} < 1$), implying that $\sigma_{00}^* < \sigma_1^* = \sigma_{11}^* < \sigma_{10}^*$, the agents can be classified into four categories according to their responses to the absence of bonus in the first period (see Figure 2):

1. "*Pessimists*": $\sigma < \sigma_{00}^*$: the agent does not work in the first period and works in the second only if offered bonus b^*.

2. "*Strategic Idlers*": $\sigma_{00}^* < \sigma < \sigma_1^*$: the agent does not work in the first period but works in the second with or without a bonus.

3. "*Fragiles*": $\sigma_1^* < \sigma < \sigma_{10}^*$: the agent works in the first period; he works in the second only if he is offered bonus b^* or if he was successful in the first period.

4. "*Enthusiasts*": $\sigma_{10}^* < \sigma$: the agent works in both periods regardless of the bonuses offered and of successfulness of the first-period effort.

Proposition 1 *i)There exist* $\bar{\theta}_H \in [c/V, 1)$ *and a continuous decreasing function* $\bar{f}(\cdot)$ *with values in* $(0,1)$ *such that if* $\theta_H > \bar{\theta}_H$ *and* $f_H < \bar{f}(\theta_H)$, *there exists a unique equilibrium satisfying Assumptions 4 and 5.*

ii)This equilibrium is semi-separating in the first period and in the subgames following (after no bonus offer in the first period) an unsuccessful effort and shirking. There is pooling in the subgame following success (after no bonus offer): $x_{11}^* = 1, \sigma_{10}^* = \sigma_1^*$. *Bonus* b^* *is given to the low-type agent in the second period if it was given in the first. The equilibrium is characterized by six parameters,* $x_1^*, x_{00}^*, x_{10}^*$ *and* $\sigma_1^*, \sigma_{00}^*, \sigma_{10}^*$, *determined by six equations*

$$G_L(\sigma_1^*) = \alpha \tag{6}$$

$$l(\sigma_1^*) = x_1^* \frac{1 + \theta_L - x_{00}^*}{\theta_H} A \tag{7}$$

$$G_L(\sigma_{00}^*) = \alpha^2 \tag{8}$$

$$l(\sigma_{00}^*) = x_1^* x_{00}^* A \tag{9}$$

$$G_L(\sigma_{10}^*) = \alpha(2 - \alpha) \tag{10}$$

$$l(\sigma_{10}^*) = \frac{1 - \theta_L}{1 - \theta_H} x_1^* x_{10}^* A \tag{11}$$

iii)In this equilibrium a "double monotonicity" property of bonuses holds:

- *In both periods the bonus given to the low-type agent (weakly) exceeds the bonus given to the high-type agent.*
- *The bonus offered in the second period (weakly) exceeds the bonus offered in the first period.*

As in the static case, for semi-separation to occur in the first period the probability of the agent having the low-type, f_L, must be non-trivial. A high probability of success for the talented agent, θ_H, is needed to make a failure sufficiently detrimental to the agent's self-confidence; otherwise, there could be complete pooling in Case B2 – low-type agents would never be offered bonus b^* after a failure in the first period.

Equations (6), (8) and (10) express the principals' indifference between giving no bonus and giving b^*. In particular, (8) and (10) take into account that in the second period the principal's beliefs about the agent's self-confidence σ are conditioned on the first-period choice of effort.

Equations (7), (9) and (11) describe the agent's reactions; (9) and (11) differ from (2) in the static model in that the agent's beliefs are properly adjusted – the term $\frac{1-\theta_L}{1-\theta_H}$ in (11) reflects discouragement from failure, $x_1^* x_{00}^*$ and $x_1^* x_{10}^*$ give the probabilities of getting no bonus offers in both periods.

The agent's reaction function $\sigma_1^*(\cdot)$ described in (7) reflects both the ratchet and the learning aspects of the agent's decision, so the overall benefit or loss from working are not tantamount to those in the single-period model. Learning means that the agent will adjust his second-period decision: if again offered no bonus in the second period, he will work after success but shirk after failure. Hence, the net expected loss (conditional on $\theta = \theta_L$) from choosing to work in the first period rather than in the second is $(1+\theta_L - x_{00}^*)(c - \theta_L V)$: the agent will suffer it in the first period and with probability θ_L in the second (after success), but would also suffer it with probability x_{00}^* in the second if abstained from working in the first. Similarly, the net expected gain (conditional on $\theta = \theta_H$) from working in the first period is $\theta_H(\theta_H V - c)$. Note that the probability that for given x_1^* the agent shirks in the first period when offered no bonus decreases with x_{00}^*: the smaller the chance to get b^* in the second period, the smaller the scope of the ratchet effect. The probability that the agent will exert effort (given x_1^*) also increases with θ_H faster than in the static case (remember that A decreases in θ_H): the higher is θ_H, the larger is the probability that the high-type agent will succeed in the first period and take the correct decision to work in the second. The higher is θ_L, however, the lower is the information value of working in the first period (the higher is the probability that the low-type agent will succeed in the first period and take the incorrect decision to work in the second), so the impact of an increase in θ_L on the probability of effort is ambiguous, because an increase in the direct benefit from working (A is an increasing function of θ_L) and a corresponding decrease in the benefit of shirking (b^* is a decreasing function of θ_L) counteract the negative information effect.

3.4 Properties of the Semi-Separating Equilibrium

All results in this subsection are obtained under the assumptions of Proposition 1, that is when the equilibrium is semi-separating (i.e. θ_H and f_L are large enough).

Comparative statics. The following results are easily obtained from equations (6) –(11):

1. An increase in the share of the low-type agents, f_L, does not alter the agent's reaction ($\sigma_1^*, \sigma_{00}^*$ and σ_{10}^*), but increases the probability of getting a bonus by the low type agent in the first period, $1 - x_1$. The second-period policy parameters x_{00} and x_{10} are not affected.

2. An increase in the probability of success for the high-type agent, θ_H, means a lower probability of getting a bonus for the weak type in the first period (x_1

increases), but a higher probability in the second conditional on no effort or failure in the first task: x_{00} and x_{10} decrease. The agent's strategy parameters $\sigma_1^*, \sigma_{00}^*$ and σ_{10}^* are unaltered.

3. In contrast, an increase in the probability of success for the low type, θ_L, increases the probability of effort when no bonus is offered ($\sigma_1^*, \sigma_{00}^*$ and σ_{10}^* go down) because the bonus required to induce effort, b^*, becomes less costly. The impact on the principals' strategies x_1, x_{00}, x_{10} is ambiguous.

4. An increase in the cost of effort c makes bonus b^* more costly, so $\sigma_1^*, \sigma_{00}^*$ and σ_{10}^* increase. The impact on the principals' policies x_1, x_{00}, x_{10} is ambiguous.

5. When the principals' benefit W increases, α, the relative cost of paying b^*, goes down so $\sigma_1^*, \sigma_{00}^*$ and σ_{10}^* fall as well, thus increasing the probability of effort without a bonus. Moreover, bonuses are more frequent in the first period: x_1 decreases. The effect on x_{00} and x_{10} is ambiguous.

6. Finally, an increase in the agent's payoff V makes bonus b^* less costly, so $\sigma_1^*, \sigma_{00}^*$ and σ_{10}^* decrease. The impact of the principal's policy parameters x_1, x_{00} and x_{10} is ambiguous.

Average efforts. When does the agent work more – in the first period or in the second? The ratchet effect produces "strategic idlers" who work in the second period without a bonus but not in the first. On the other hand, because of discouragement by failure and also because of the disappearance of the incentive to learn in the second period, there are "fragile" agents who work in the first period without a bonus but do not work in the second after a failure unless promised b^*. The following lemma shows that the first effect dominates the second when the high-type agents are talented enough (θ_H is high).

Lemma 3 *i) When θ_H is large enough, the average effort of each type of agent is larger in the second period: $\bar{e}_2^H > \bar{e}_1^H, \bar{e}_2^L > \bar{e}_1^L$.*

ii) The average effort of the high-type agent in the first period is the same as in the static model: $\bar{e}_1^H = \bar{e}^H$. The average effort of the low-type agent in the first period will be lower than in the static model: $\bar{e}_1^L < \bar{e}^L$ if $\theta_H - \theta_L$ is large enough.

Welfare. In the case of partial separation, the principal's welfare in the static model is lower than it would be under symmetric information: her payoff when the agent has the low type is the same, $U_L^P = \theta_L(W - b^*)$, but the payoff from the high-type agent $U_H^P = \theta_H(1 - G_H(\sigma^*))W$ is lower than it would be in the symmetric information benchmark ($\theta_H W$). The reason is simple: pooling spoils the signal sent to the high-type agent who then does not always work.

When there are two principals, the first one gets the same payoff as she would get in the static model. However, when θ_H is large enough, the second principal has a higher welfare:

Lemma 4 *i)When the agent has the low type, the second principal is better off than the first:*

$$U_L^{P2} > U_L^{P1} = U_L^P.$$

ii)If θ_H is large enough and the agent has the high type, the second principal is better off than the first:

$$U_H^{P2} > U_H^{P1} = U_H^P.$$

When the agent has the low type, the second principal strictly gains (gets more than $\theta_L(W-b^*)$ in expectation) after success of the agent who was not given a bonus in the first period. When the agent has the high type, if θ_H is large the loss from discouragement of "fragile" agents is smaller than the gain from "strategic idlers" who start working – hence, the second part of the lemma.

4 The Model With A Permanent Principal

Suppose now that the agent faces the same principal in both periods. The timing of events and the information sets of players are the same as in the previous section. I also assume that the principal has limited commitment ability: at date 1 she can offer a bonus contingent only on date 1 performance.

The analysis of the continuation equilibria in the previous section translates literally to the permanent principal vesrion. In contrast, the long-term principal's problem in the first period is more complex: she has to take into account the impact of her policy on *the agent's self-confidence after the first period* whereas a transient principal cares only about *the agent's behavior in the first period*. The following proposition shows that there exists an equilibrium with the same structure as in the two-principal version. Unfortunately, proving its uniqueness (under appropriate assumptions, say an extended version of NWBR criterion) is problematic.

Proposition 2 *i)There exist $\tilde{\theta}_H \in [c/V, 1)$ and a continuous decreasing function $\tilde{f}(\cdot)$ with values in $(0,1)$ such that if $\theta_H > \hat{\theta}_H$ and $f_H < \tilde{f}(\theta_H)$, there exists an equilibrium satisfying Assumptions 4 and 5 in the second-period subgames.*

ii)This equilibrium is semi-separating in the first period and in the subgames following (after no bonus offer in the first period) an unsuccessful effort and shirking. There is pooling in the subgame following success (after no bonus offer): $\tilde{x}_{11} = 1, \tilde{\sigma}_{10} = \tilde{\sigma}_1$. Bonus b^* is given to the low-type agent in the second period if it was given in the first. The equilibrium is characterized by six parameters, $\tilde{x}_1, \tilde{x}_{00}, \tilde{x}_{10}$ and $\tilde{\sigma}_1, \tilde{\sigma}_{00}, \tilde{\sigma}_{10}$, determined by six equations

$$G_L(\tilde{\sigma}_1) = \frac{\alpha(1+\theta_L)}{1+\alpha\theta_L}, \tag{12}$$

$$l(\tilde{\sigma}_1) = \tilde{x}_1 \frac{1+\theta_L - \tilde{x}_{00}}{\theta_H} A, \tag{13}$$

$$G_L(\tilde{\sigma}_{00}) = \frac{\alpha^2(1+\theta_L)}{1+\alpha\theta_L}, \tag{14}$$

$$l(\tilde{\sigma}_{00}) = \tilde{x}_1 \tilde{x}_{00} A, \tag{15}$$

$$G_L(\tilde{\sigma}_{10}) = \frac{\alpha(2-\alpha+\theta_L)}{1+\theta_L\alpha}, \tag{16}$$

$$l(\tilde{\sigma}_{10}) = \frac{1-\theta_L}{1-\theta_H} \tilde{x}_1 \tilde{x}_{10} A. \tag{17}$$

iii)In this equilibrium a "double monotonicity" property of bonuses holds:

- *In both periods the bonus given to the low-type agent (weakly) exceeds the bonus given to the high-type agent.*
- *The bonus offered in the second period (weakly) exceeds the bonus offered in the first period.*

When there are two principals, there exists an externality imposed by the first of them on the second: when the first principal offers a type-revealing bonus b^* in the first period, she creates the agent's addiction to rewards and the second principal is bound to pay b^* again. In the model with one permanent principal this externality is internalized. A long-lasting principal gains more than a transient one by not offering a bonus to the weak agent in the first period: if the agent succeeds, the principal gets an expected payoff $\theta_L W$ in the second period instead of $\theta_L(W-b^*)$ which she would get if she offered b^* in the first period.[26] Hence, the new threshold level $\tilde{\sigma}_1$ is higher than σ_1^*.

All the following results show the properties of the equilibrium specified in Proposition 2.

Proposition 3 *A long-lasting principal is less likely to reward the agent in the first period than a transient one:* $1-\tilde{x}_1 < 1-x_1^*$.

[26]In the case of partial separation, if the agent fails or does not even try, the principal gets an expected payoff $\theta_L(W-b^*)$ in the second period, the same as she would get if she offered b^* from the beginning.

Average efforts. We can now compare the average efforts in the settings with transitory principals and with a permanent one.

Proposition 4 *If θ_H is large enough,*

i) The average effort increases in the second period for each type of agent: $\tilde{e}_2^H > \tilde{e}_1^H, \tilde{e}_2^L > \tilde{e}_1^L$.

ii) The average effort in each period is lower for each type of agent when there is a permanent principal: $\tilde{e}_1^H < \bar{e}_1^H, \tilde{e}_1^L < \bar{e}_1^L; \tilde{e}_2^H < \bar{e}_2^H, \tilde{e}_2^L < \bar{e}_2^L$.

The first part of the proposition is the same as in the model with two principals. The fact that there is less effort in the first period with a long-lasting principal is a straightforward implication of the principal's increased concern about paying bonuses in the first period. This concern increases all thresholds: $\tilde{\sigma}_1 > \sigma_1^*, \tilde{\sigma}_{00} > \sigma_{00}^*, \tilde{\sigma}_{10} > \sigma_{10}^*$,which results in a lower effort in the second period as well.

Welfare. Since there is less effort with a permanent principal, the total welfare is lower in this case. However, when facing the low-type agent, the long-lasting principal internalizes the addiction-creating externality which existed when there were two principals. Paradoxically, she does not gain from this:

Proposition 5 *Internalization backfires: for each type of agent, in the case of semi-separation the welfare of the permanent principal is lower than the total welfare of two transient ones:*

$$\begin{aligned}\tilde{U}_L^P &< U_L^{P1} + U_L^{P2}, \\ \tilde{U}_H^P &< U_H^{P1} + U_H^{P2}.\end{aligned}$$

The results of Proposition 5 look surprising: why can't the permanent principal achieve the same welfare as the transient ones by mimicing their begavior? The answer is that this is not credible: were the permanent principal to stick to the same policy as the transient ones, and, consequently, were the agent to have the same believes after being offered $b_1 = 0$ as in the two-principal situation, the principal would like to deviate and offer $b_1 = 0$ more frequently. The agent is not a fool, though, and the principal's reluctance to offer rewards in the first period backfires. The fact that the principal(s) lose(s) from a long-term relationship when the agent has the high type is quite intuitive: there is a larger probability of pooling of the low-type agent with the high type, so the latter works less. However, it is quite striking that when the agent has the low type, the ability to control creation of addiction harms the principal (at least, in the case of semi-separation).

It is easy to show that the high-type agent gets a lower expected surplus when he faces a long-lasting principal:

Lemma 5 *The high-type agent is worse off when he faces a permanent principal:*

$$\tilde{U}_H^{A1} + \tilde{U}_H^{A2} < U_H^{A1} + U_H^{A2}.$$

The impact on the low-type agent is ambiguous: he may obtain either a higher or a lower surplus in a long-term relationship. Example 1 in Appendix B shows that both possibilities can be realized. Intuitively, the agent will get a higher surplus in a long-term relationship if the difference in the principal's policy $(\tilde{x}_1 - x_1^*)$ is small relative to the difference in the agent's reaction $(G_L(\tilde{\sigma}_1) - G_L(\sigma_1^*))$.

Taking a broader view, one can ask how one- or two-principal arrangements affect the agent's ex post self-confidence. Complete analysis is too cumbersome, but two things are clear. Firstly, self-confidence of the high-type agent is lower with a permanent principal, because more low-type agents are pooled with him. Secondly, in a long-term relationship bonuses are less frequent so there are fewer completely depressed agents with the lowest possible beliefs (probability 1 of θ_L).

Within the analyzed principal-agent interaction taken in isolation, the agent gains from having an adequate self-image even when it is unfavorable. Put differently, for the agent there is no cost of getting addicted to rewards. However, there are many circumstances in which the agent would be better off avoiding bad news, even if at some short-run cost (e.g., excessive effort as in this model). One example is when θ reflects some combination of the agent's abilities and task characteristics. Then, an offer of a high bonus, even if meant only to reflect poor fitness for the project, may still cause an undue loss of the agent's confidence in his abilities and backfire when the agent faces a task to which he is more apt. Alternatively, the principal(s) can have a biased perception of the agent or the task, and this erroneous perception may be transmitted to the agent: e.g., the parents who have had hard times at school may too often use rewards (or punishments) to stimulate their children to learn and get high grades. Finally, for several reasons (e.g. to overcome time-inconsistency) it may be optimal for the agent to have a somewhat overoptimistic self-image[27].

5 Alternative Specifications

This section investigates the robustness of my results. Firstly, I will show that the non-observability of the agent's first-period effort does not change the structure of equilibria: in particular, the "double monotonicity" property in Propositions 1 and 2 still holds.

Then, an alternative formulation of the model, in which the agent knows his ability θ but is uncertain about the payoff V, will be examined. An important feature of the previous version of the model was a mild information acquisition – the agent was receiving a natural endogenous signal (success or failure in the first task), which provided a useful, but not definitive information about his unknown ability. With the uncertain payoff I consider two polar cases – one, in which the agent immediately learns (without noise) his gain V if he achieves it, is studied

[27] See Bénabou and Tirole (2002) for an extensive analysis.

in Section 5.2. As we shall see, results may be reversed with this extreme form of learning: the permanent principal may offer a high bonus to the high-valuation agent in order to urge him to learn his payoff in the first period. Section 5.3 examines a situation in which the agent learns nothing until the end of the game, so the learning motive is completely eliminated.[28] (With unknown cost c analysis would be similar to the one with unknown V.)

Finally, in Section 5.4 I extend the space of contracts available to the principal by allowing for lump-sum payments. As will be seen, lump-sum payments are useless under some (not too restrictive) assumptions.

In this section results are presented informally, with more precise statements relegated to Appendix A.

5.1 Non-observability of Effort

The model is the same as in Sections 3.1 and 4 with one exception: the second-period principal does not observe the agent's first-period effort. Then, when observing a failure in the first period, the principal has to guess whether it came from bad luck or the agent did not even try – the model becomes one of "hidden knowledge".

The following proposition shows that this complication is minor: in the subgame following a failure (caused by either bad luck or shirking) the principal behaves as if she believed that the agent received a "virtual" signal, the distribution of which satisfies the MLRP.

Proposition 6 *i)There exist $\hat{\theta}_H \in [c/V, 1)$ and a continuous decreasing function $\hat{f}(\cdot)$ with values in $(0, 1)$ such that if $\theta_H > \hat{\theta}_H$ and $f_H < \hat{f}(\theta_H)$, there exists an equilibrium satisfying Assumptions 4 and 5 in the second-period subgames.*

ii)This equilibrium is semi-separating in the first period and in the subgame following (after no bonus offer in the first period) an unsuccessful effort or shirking. There is pooling in the subgame following success (after no bonus offer): $\hat{x}_{11} = 1, \hat{\sigma}_{10} = \hat{\sigma}_1$. Bonus b^* is given to the low-type agent in the second period if it was given in the first.

iii)Proposition 2 describing a semi-separating equilibrium in the model with a permanent principal extends to this setting in a similar way.

5.2 Unknown Valuation Learned After Success

In this model the agent's valuation can assume two values: V_L with probability f_L and $V_H > V_L$ with probability f_H. In case of success in the first period, the agent

[28] The last assumption may be appropriate in education, where a student can sometimes assess the value of particular knowledge only after completing the whole program. In the business context, the benefits for one's career from working (successfully) on a certain project may become evident not until several such projects are implemented; or the project may be a multistage one.

learns V before the second period. Otherwise, the model is the same as before.[29]

Transitory principal. As in the model with unknown θ, both principals never give a bonus to the high-type agent and randomize between 0 and b^* for the low-type agent unless he knows his type in which case he gets b^*.

The continuation subgame following success in the first period is different from the one in section 3.1: the asymmetry of information about the agent's valuation disappears and so the agent is offered the full-information bonus in the second period (0 for the high type and b^* for the low).

Since failure no longer has a negative informational content, one could expect the category of "fragile" agents to vanish. Yet, as Lemma 11 (in Appendix A) shows, this class of agents may persist if there is sufficiently strong additional incentive to work in the first period (as compared to the second) provided by the value of learning. The agent strictly gains from achieving the symmetry of information in the second period: the low-type agent will be sure to get an adequate bonus and avoid possible losses from excessive optimism, while the high-type agent will be sure to make the right decision and work in response to no bonus offer. However, for learning to be motivating, the probability of acquiring information, that is, the probability of success θ must be large enough.

Permanent principal. With the agent's ability as the uncertain parameter, equilibria in the models with one and two principals had the same structure and differed only quantitatively. With unknown valuation the situation is different: an equilibrium where the principal never gives a bonus to the high-type agent and randomizes between no bonus and b^* for the low-type agent may not exist (Example 2 in Appendix B demonstrates the validity of this claim). The reason is that now in case of success the agent gets hard evidence about his type and the principal wants to stimulate the high-type agent to collect this evidence (exert effort) whereas, in contrast, she is concerned about the low-type agent learning his payoff. Addiction fails to develop in this case.

5.3 Unknown Valuation Learned After the Second Period

In this setting the agent learns nothing about his unknown payoff V, save for what he can deduce from the principals' policy. With all learning eliminated, "fragile" agents disappear: neither there is a discouragement effect, for a failure in the first period has no information impact, nor there is any difference in the value of success across periods because of the absence of learning considerations. Apart from that, the equilibrium is similar to the one with unknown ability θ: the principal never

[29] The assumptions are properly modified: Assumption 1 is now $\theta V_L < c < \theta V_H$; the type-revealing bonus $b^* = c/\theta - V_L$. High σ is good news about V in the sense of MLRP (Assumption 3).

offers a bonus to the high-valuation agent and randomizes between no bonus and b^* for the low type (provided that the share of low-valuation agents f_L is large enough so that there is partial separation in the first period). Equilibrium parameters are given in Lemma 12 in Appendix A.

In this model the agent's "ratcheting" is not mitigated by the incentive to acquire information. Then, in the absence of the principal's ratcheting (i.e. in the model with two transient principals) in equilibrium there is less pooling than in the static model. Consequently, there is less effort in the first period. This clear-cut prediction contrasts with the ambiguity which existed when both the agent's ratchet and learning effects were present (Lemma 3).

5.4 Lump-sum transfers

I have ignored so far the possibility of the principal's signalling the agent's type through lump-sum transfers. There are several exogenous reasons for the principal not to use lump-sum transfers: she may be simply credit-constrained, or prefer not to spend money gained elsewhere on this particular interaction as the theory of "mental accounting" (e.g. Thaler (2000)) would predict in some situations.

More importantly, the scope for lump-sum payments is quite limited in this model. Since the agent's participation is not at stake and because of his limited liability, signalling through "burning money" is the only use of lump-sum transfers. An important remark is that equilibria which have been studied in the settings with a permanent and transient principals remain equilibria if lump-sum payments are introduced: to support them, the agent's perception of the task out of equilibrium should not be affected by the level of the uncontingent wage. However, allowing for lump-sum transfers may result in less pooling. In particular, under some conditions a separating equilibrium in a one-shot relationship satisfies the NWBR criterion.

In this section I shall analyze the static model with lump-sum payments and then extend it to the two-period setting with two principals. The main lesson is that under some reasonable restrictions on parameters lump-sum transfers will not be used in equilibrium and all the analysis performed so far remains valid. Unfortunately, I do not have sharp results for the case of a permanent principal.

5.4.1 The static model

The principal now offers contracts (a, b) consisting of an unconditional wage $a \geq 0$ and a bonus payment $b \geq 0$. I stick to assumptions 1-4 (in particular, focusing on equilibria satisfying the NWBR criterion) with one exception: the likelihood ratio $l(\sigma)$ may now be bounded away from 0; however, to avoid additional complications and without losing much insight I keep assuming that $l(\sigma) \to \infty$ as $\sigma \to \infty$.

Leaving the formal development to the Appendix, I will present here the main results.

As the following proposition shows, the equilibria of the static game depend on three crucial parameters. The first is $l_0 = l(0)$, characterizing informativeness of the lower range of the agent's signal. The second is $A = \frac{f_L}{f_H}\frac{c-\theta_L V}{\theta_H V - c}$, which measures the agent's willingness to work: in the absence of informative signals the agent would work if $A \leq 1$ and shirk otherwise. Note that A is a decreasing function of f_H, with $A(f_H) \to \infty$ as $f_H \to 0$ and $A(1) = 0$, so there exists $\bar{f}_H$ such that $A(\bar{f}_H) = \frac{\theta_L}{\theta_H}$. The third parameter is the principal's payoff W. When $l_0 < \frac{\theta_L}{\theta_H}$, there exists $\bar{W}$ such that $\bar{W} = \frac{b^*}{G_L(\bar{\sigma})}$ and $l(\bar{\sigma}) = \frac{\theta_L}{\theta_H}$. Finally, when $l_0 < \frac{\theta_L}{\theta_H}$ and $W > \bar{W}$, there exists $\tilde{f}_H(W) > \bar{f}_H$ such that $A(\tilde{f}_H(W)) = l(\tilde{\sigma})$ and $G_L(\tilde{\sigma}) = \frac{b^*}{W}$.

Proposition 7 *i) When $l_0 < \frac{\theta_L}{\theta_H}$ generically there is a unique equilibrium (satisfying NWBR)*[30]*:*

A) When $W > \bar{W}$, there are three different types of equilibrium:

∘ *When $(l_0, f_H) \in \mathcal{R}_1 = \{(l_0, f_H) : A(f_H) < l_0 < \frac{\theta_L}{\theta_H}\}$, the equilibrium is pooling: contract $(0,0)$ is offered and the agent always works in response.*

∘ *When $(l_0, f_H) \in \mathcal{R}_{2A} = \{(l_0, f_H) : l_0 < A(f_H), f_H > \tilde{f}_H(W)\}$, the equilibrium is pooling: contract $(0,0)$ is offered and the agent works if $\sigma \geq \tilde{\sigma}$, where $\tilde{\sigma}$ is determined by $l(\tilde{\sigma}) = A(f_H)$.*

∘ *When $(l_0, f_H) \in \mathcal{R}_{3A} = \{(l_0, f_H) : l_0 < A(f_H), f_H < \tilde{f}_H(W)\}$, the equilibrium is semi-separating: contract $(0,0)$ is always offered when $\theta = \theta_H$ and is offered with probability $\tilde{x}$ when $\theta = \theta_L$ and contract $(0, b^*)$ is offered with probability $1 - \tilde{x}$ when $\theta = \theta_L$. When offered $(0,0)$, the agent works if $\sigma \geq \tilde{\sigma}$. Equilibrium parameters $\tilde{\sigma}$ and $\tilde{x}$ are determined by $G_L(\tilde{\sigma}) = \frac{b^*}{W}$ and $l(\tilde{\sigma}) = \tilde{x}A(f_H)$.*

B) When $W < \bar{W}$, there are three different types of equilibrium:

∘ *When $(l_0, f_H) \in \mathcal{R}_1 = \{(l_0, f_H) : A(f_H) < l_0 < \frac{\theta_L}{\theta_H}\}$, the equilibrium is pooling: contract $(0,0)$ is offered and the agent always works in response.*

∘ *When $(l_0, f_H) \in \mathcal{R}_{2B} = \{(l_0, f_H) : l_0 < A(f_H), f_H > \bar{f}_H\}$, the equilibrium is pooling: contract $(0,0)$ is offered and the agent works if $\sigma \geq \tilde{\sigma}$, where $\tilde{\sigma}$ is determined by $l(\tilde{\sigma}) = A(f_H)$.*

∘ *When $(l_0, f_H) \in \mathcal{R}_{3B} = \{(l_0, f_H) : l_0 < A(f_H), f_H < \bar{f}_H\}$, the equilibrium is semi-separating: contract $(\tilde{a}, 0)$ is always offered when $\theta = \theta_H$ and is offered with probability $\tilde{x}$ when $\theta = \theta_L$ and contract $(0, b^*)$ is offered with probability $1 - \tilde{x}$ when $\theta = \theta_L$. When offered $(0,0)$ the agent works if $\sigma \geq \tilde{\sigma}$. Equilibrium parameters $\tilde{a}, \tilde{\sigma}$ and $\tilde{x}$ are determined by $l(\tilde{\sigma}) = \frac{\theta_L}{\theta_H}, l(\tilde{\sigma}) = \tilde{x}A(f_H)$ and $\tilde{a} = b^* - G_L(\tilde{\sigma})W$.*

ii) When $l_0 > \frac{\theta_L}{\theta_H}$ there is a continuum of equilibria satisfying NWBR (in each of them the agent always works):

∘ *When $(l_0, f_H) \in \mathcal{R}_4 = \{(l_0, f_H) : \frac{\theta_L}{\theta_H} < l_0 < A(f_H)\}$, the set of equilibria contains*

∘ *a separating equilibrium with $(a^*, 0)$ offered when $\theta = \theta_H$ and $(0, b^*)$ offered when $\theta = \theta_L$;*

[30] Equilibrium may not be unique on the set of parameters of measure zero – on the boundaries of the regions described below. For brevity we omit the analysis of these non-generic cases.

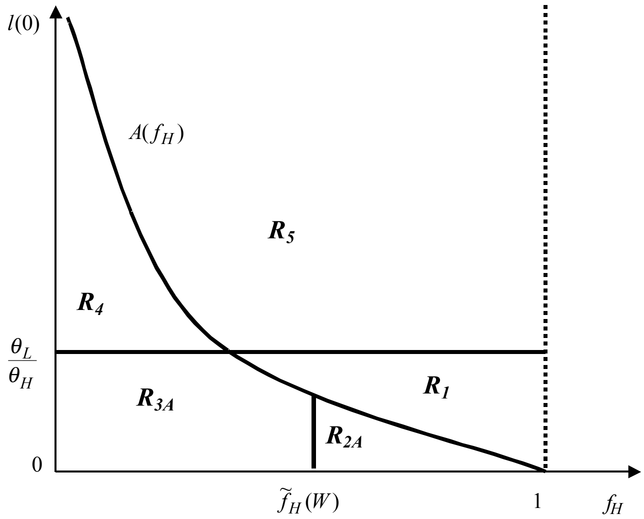

Figure 3:

○ *a continuum of semi-separating equilibria, with* $(a^*, 0)$ *always offered when* $\theta = \theta_H$ *and with probability* $x \in [0, l_0/A]$ *when* $\theta = \theta_L$ *and* $(0, b^*)$ *offered with probability* $1 - x$ *when* $\theta = \theta_L$.

○ *When* $(l_0, f_H) \in \mathcal{R}_5 = \{(l_0, f_H) : l_0 > A(f_H), l_0 > \frac{\theta_L}{\theta_H}\}$, *the set of equilibria contains all equilibria of region* $\mathcal{R}_4$ *(with* $x \in [0, 1]$ *for the semi-separating equilibrium) and*

○ *a continuum of pooling equilibria with contract* $(\tilde{a}, 0)$, *where* $\tilde{a} \in [0, a^*]$.

Figures 3 and 4 illustrate Proposition 7.

Corollary 4 *When* $l_0 < \frac{\theta_L}{\theta_H}$ *and* $W > \bar{W}$, *the unique equilibrium (satisfying NWBR) of the model with lump-sum transfers is the same as in the model without lump-sum transfers.*

The main implication of Proposition 7 is that when the agent's signal is sufficiently informative and the principal's payoff is large enough there is no room for "burning money" strategies. On the other hand, B&T have shown that when $l_0 > \frac{\theta_L}{\theta_H}$ (which, given MLRP, is equivalent to their limited informativeness assumption $\theta_H G_H(\sigma) > \theta_L G_L(\sigma)$ for all σ) separating equilibrium satisfies the NWBR refinement. Which assumption is more reasonable, $l_0 > \frac{\theta_L}{\theta_H}$ or $l_0 < \frac{\theta_L}{\theta_H}$, depends upon the application. Since σ often reflects past experience, one can assume, for example, that σ is a continuous piecewise-linear extrapolation of a discrete signal ν, which is obtained in the following way. The agent has performed N attempts to fulfill a similar task in the past, and ν is the share of successful attempts. Then

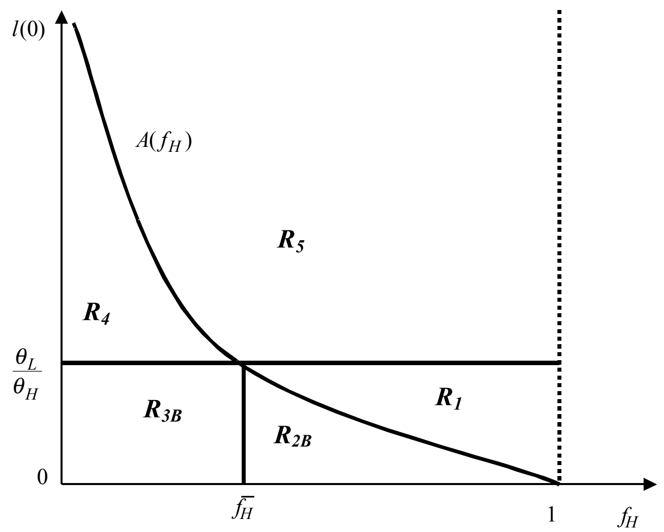

Figure 4:

$l(0) = (\frac{1-\theta_H}{1-\theta_L})^N$, where $l(0) = \frac{\Pr\{\nu=0|\theta=\theta_H\}}{\Pr\{\nu=0|\theta=\theta_L\}}$ and the signal ν will be sufficiently informative ($l(0) < \frac{\theta_L}{\theta_H}$) when the agent has had much experience with similar tasks before (N is large enough) but ν may have limited informativeness ($l(0) > \frac{\theta_L}{\theta_H}$) when N is small and both θ_H and θ_L are small enough.

5.4.2 The two-period model

My program here is modest: not attempting a complete analysis I shall show that lump-sum transfers will not be used under some (reasonable) restrictions on parameters in the model with two principals.

Proposition 8 *Assume that W and θ_H are large enough, l_0 and f_H are small enough. Then an equilibrium described in Proposition 1 satisfies the NWBR criterion in each period.*

6 Concluding Remarks

The paper first shows that rewards are addictive: once offered a bonus, the agent keeps demanding it whenever he is expected to exert effort. Concerned by addiction, a principal, who is going to deal with the agent in the future, adopts a more conservative reward policy than a transient principal.

For the addiction phenomenon to arise, a sorting condition, which makes the principal more prone to offer bonuses to less able agents, must be satisfied in the early period of the multi-stage relationship. I show that this condition is satisfied in a model, in which the agent is uncertain about his chances to succeed and, apart from the principal's bonus offers, he observes his interim performance. However, the analysis of another version of the model (Section 5.2), where success in the first task immediately discloses to the agent his ex ante unknown payoff, leads to different conclusions. The sorting condition may be reversed in the first period because, if the agent's valuation is high, the principal is so eager to convince the agent to exert effort and learn his payoff that she may promise a high bonus. In this case behavior of a long-lasting principal is radically different from the behavior of a transient one, who would not bother about the agent's information acquisition and would keep paying lower bonuses when the payoff is higher.

There are several interesting questions to be answered in future research. Firstly, it would be interesting to see how the equilibrium changes and what consequences for the parties' welfare this entails when the second transient principal does not observe what was happening in the first period as clearly she does in this model (e.g. the first principal's policy may be unobservable). Another interesting question is whether in the dynamic context the agent gains from the presence of a principal (who is useless technologically, but is informed and can provide incentives). As it was proved, the agent gains in the static model, but it is not clear that the result extends to the dynamic setting since gaming with the principal (ratchet effect) may dissipate the gains. Yet another research project is to endogenize the relationship formation. For example, the agent may choose whether to stay with the same principal or to go to a new one. Alternatively, the principal may decide whether to keep a permanent relationship with the agent or to delegate it for some time to someone else (e.g. leave the child to grand-parents).

Experiments concerning the effect of rewards on intrinsic motivation already abound in economics and in social psychology (see, for example, a survey of Frey and Jegen (2001)). This paper makes several clear-cut predictions, which could also be tested in a laboratory experiment. Understanding an artificial economic environment, corresponding to the model considered in the paper, and computing equilibria demands a good deal of rationality, and it would be particularly interesting whether gut feelings and heuristics would bring the subject to adopt behaviors approximating the ones discussed in the paper.

7 Appendix A

7.1 The model with transitory principals

Proof of Lemma ??. When the agent acts without a principal, his ex ante expected welfare is

$$U_A^{alone} = f_H(1 - G_H(\tilde{\sigma}^{alone}))(\theta_H V - c) + f_L(1 - G_L(\tilde{\sigma}^{alone}))(\theta_L V - c),$$

where $\tilde{\sigma}^{alone}$ is a threshold level of σ above which the agent decides to work (if there is no principal) determined by $l(\tilde{\sigma}^{alone}) = A$. With a principal the expected welfare is

$$U_A^{withP} = f_H(1 - G_H(\sigma^*))(\theta_H V - c) + f_L x^*(1 - G_L(\sigma^*))(\theta_L V - c),$$

where σ^* and x^* are defined in (2) and (3). Hence

$$\begin{aligned} U_A^{withP} - U_A^{alone} &= [f_H(G_H(\tilde{\sigma}^{alone}) - G_H(\sigma^*))(\theta_H V - c) \\ &\quad -f_L x^*(G_L(\tilde{\sigma}^{alone}) - G_L(\sigma^*))(c - \theta_L V)] \\ &\quad +f_L(1 - x^*)(1 - G_L(\tilde{\sigma}^{alone}))(c - \theta_L V). \end{aligned} \tag{18}$$

The expression in square brackets in (18) is positive since $\tilde{\sigma}^{alone} > \sigma^*$ so[31]

$$\frac{G_H(\tilde{\sigma}^{alone}) - G_H(\sigma^*)}{G_L(\tilde{\sigma}^{alone}) - G_L(\sigma^*)} > l(\sigma^*) = x^* \frac{f_L(c - \theta_L V)}{f_H(\theta_H V - c)}. \tag{19}$$

Since the last summand in (18) is positive, $U_A^{withP} > U_A^{alone}$. ■

Proof of Lemma 2. Consider the agent's net benefit from working in the first period when he receives a signal σ :

$$U_A^{net}(\sigma) = \hat{p}_H(\sigma)(U_A^1(\theta_H, \sigma) - U_A^0(\theta_H, \sigma)) + \hat{p}_L(\sigma)(U_A^1(\theta_L, \sigma) - U_A^2(\theta_L, \sigma))$$

where $U_A^1(\theta_i, \sigma)$ is the expected utility of the agent in both periods if he decides to work in the first period after receiving a signal σ and his true type is θ_i (similarly $U_A^0(\theta_i, \sigma)$ is the expected utility if he shirks) and $\hat{p}_H(\sigma)$ is the probability[32] that $\theta = \theta_H$ conditional on σ, $\hat{p}_L(\sigma) = 1 - \hat{p}_H(\sigma)$. Note that for the high-type agent the benefit from working in the first period, $U_A^1(\theta_H, \sigma) - U_A^0(\theta_H, \sigma)$, is positive, and for the low-type $U_A^1(\theta_L, \sigma) - U_A^2(\theta_L, \sigma)$ is negative. Moreover, these net benefits do not change when σ increases save when σ crosses one of the thresholds defining the agent's behavior in continuation subgames. When σ crosses such a threshold, $U_A^1(\theta_H, \sigma) - U_A^0(\theta_H, \sigma)$ increases and $U_A^1(\theta_L, \sigma) - U_A^0(\theta_L, \sigma)$ decreases. However, since these thresholds are determined optimally for the agent, the net effect of such a change on $U_A^{net}(\sigma)$ is either of the second order or positive. On the other hand, $\hat{p}_H(\sigma)$ strictly increases with σ and $\hat{p}_L(\sigma)$ decreases, so $U_A^{net}(\sigma)$ is increasing in σ, which proves the lemma. ■

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[31] The ineqality in (19) follows from a useful property implied by MLRP, which will be used many times in the following proofs: for any $\sigma_2 > \sigma_1$

$$l(\sigma_1) < \frac{G_H(\sigma_2) - G_H(\sigma_1)}{G_L(\sigma_2) - G_L(\sigma_1)} < l(\sigma_2).$$

To obtain, for example, the first inequality, just integrate with respect to σ from σ_1 to σ_2 inequality $g_H(\sigma) > g_L(\sigma)l(\sigma_1)$, which holds for all $\sigma > \sigma_1$.

[32] $\hat{p}_H(\sigma) = \frac{f_H x_1^H g_H(\sigma)}{f_H x_1^H g_H(\sigma) + f_L x_1^L g_L(\sigma)}$ if the agent receives bonus b_1 in the first period, which is given with probability x_1^H to the high-type agent and with probability x_1^L to the low-type one.

Lemma 6 *The equilibrium strategies x^*_{11} and σ^*_{11} are defined by*

$$\sigma^*_{11} = \max\left\{ l^{-1}(x^*_1 x^*_{11} \frac{\theta_L}{\theta_H} A), \sigma^*_1 \right\} \tag{20}$$

and either

$$G_L(\sigma^*_{11}) = 1 - (1-\alpha)(1 - G_L(\sigma^*_1)) \tag{21}$$

or

$$G_L(\sigma^*_{11}) < 1 - (1-\alpha)(1 - G_L(\sigma^*_1)) \quad \text{and } x^*_{11} = 1. \tag{22}$$

Proof. The principal offers no bonus to the weak agent with probability $x^*_{11} \in (0,1]$. The agent who can find himself in such a situation has received $\sigma \geq \sigma^*_1$ since he chose to work in the first period, hence $\sigma^*_{11} \geq \sigma^*_1$. The agent will work when offered no bonus in the second period if

$$\left(\frac{x^*_1 x^*_{11} \theta_L g_L(\sigma)}{\theta_H g_H(\sigma)} \right) \left(\frac{c - \theta_L V}{\theta_H V - c} \right) \leq 1. \tag{23}$$

Indeed, (23) states that the expected gain from working (which arrives when $\theta = \theta_H$) is at least as large as the expected loss from working (happening when $\theta = \theta_L$). Note that the new agent's odds of being the low type versus the high type take into account both principals' equilibrium policies ($x^*_1 x^*_{11}$ instead of x^* in Lemma 1) and the success in the first period (θ_L/θ_H term). The threshold level σ^*_{11} is the minimal signal that satisfies both inequalities $\sigma \geq \sigma^*_1$ and (23), which proves (20).

The distribution of signal σ conditional on the agent's type and effort exerted in the first period is a truncation of the distribution conditional only on the agent's type: now the principal knows that σ is above the threshold σ^*_1. For the principal to be willing to give no bonus to the low-type agent with a positive probability, his net gain from offering no bonus, $W \Pr\{\sigma \geq \sigma^*_{11}\}\theta_L - (W - b^*)\theta_L$, must be nonnegative, which means

$$\frac{1 - G_L(\sigma^*_{11})}{1 - G_L(\sigma^*_1)} \geq 1 - \alpha,$$

that is

$$G_L(\sigma^*_{11}) \leq 1 - (1-\alpha)(1 - G_L(\sigma^*_1)).$$

(21) corresponds then to the semi-separating equilibrium and (22) to the pooling one. ■

The proofs of the next two Lemmas are almost identical to that of Lemma 6 and are therefore omitted.

Lemma 7 *The equilibrium strategies* x^*_{10} *and* σ^*_{10} *are defined by*

$$\sigma^*_{10} = \max\left\{ l^{-1}(x^*_1 x^*_{10} \frac{(1-\theta_L)}{(1-\theta_H)} A), \sigma^*_1 \right\} \tag{24}$$

and either

$$G_L(\sigma^*_{10}) = 1 - (1-\alpha)(1 - G_L(\sigma^*_1)) \tag{25}$$

or

$$G_L(\sigma^*_{10}) < 1 - (1-\alpha)(1 - G_L(\sigma^*_1)) \quad \text{and } x^*_{10} = 1. \tag{26}$$

Lemma 8 *The equilibrium strategies* x^*_{00} *and* σ^*_{00} *are defined by*

$$\sigma^*_{00} = \min\left\{ l^{-1}(x^*_1 x^*_{00} A), \sigma^*_1 \right\}. \tag{27}$$

and either

$$G_L(\sigma^*_{00}) = \alpha G_L(\sigma^*_1) \tag{28}$$

or

$$G_L(\sigma^*_{00}) < \alpha G_L(\sigma^*_1) \quad \text{and } x^*_{00} = 1. \tag{29}$$

Lemma 9 *i)If in the second-period continuation equilibria the agent's reactions satisfy* $\sigma^*_{00} < \sigma^*_1 = \sigma^*_{11} < \sigma^*_{10}$*, then the equilibrium strategies* x^*_1 *and* σ^*_1*, are defined by*

$$l(\sigma^*_1) = x^*_1 \frac{1 + \theta_L - x^*_{00}}{\theta_H} A \tag{30}$$

and

$$G_L(\sigma^*_1) = \alpha \tag{31}$$

in the case of semi-separation in the first period and

$$\sigma^*_1 = l^{-1}(\frac{1 + \theta_L - x^*_{00}}{\theta_H} A) \tag{32}$$

and $x^*_1 = 1$ *in the case of pooling (if* $G_L(\sigma^*_1) < \alpha$*).*

ii)If in the second-period continuation equilibria the agent's reactions satisfy $\sigma^*_{00} < \sigma^*_1 = \sigma^*_{11} = \sigma^*_{10}$*, then the equilibrium strategies* x^*_1 *and* σ^*_1*, are defined by*

$$l(\sigma^*_1) = x^*_1 (2 - x^*_{00}) A \tag{33}$$

and

$$G_L(\sigma^*_1) = \alpha \tag{34}$$

in the case of semi-separation in the first period and

$$\sigma^*_1 = l^{-1}((2 - x^*_{00}) A) \tag{35}$$

and $x^*_1 = 1$ *in the case of pooling (if* $G_L(\sigma^*_1) < \alpha$*).*

Proof. i)(30) can be rewritten as

$$\left(\frac{x_1^* g_L(\sigma)}{g_H(\sigma)}\right)\left(\frac{(1+\theta_L - x_{00}^*)(c-\theta_L V)}{\theta_H(\theta_H V - c)}\right) \leq 1. \tag{36}$$

The first term in (36) stands for the odds of being the low-type versus the high-type agent. The agent that receives a signal which makes him indifferent between working or shirking in the first period if $b_1 = 0$, after being offered $b_2 = 0$ will choose to work after success but to shirk after failure if he has chosen to work in the first period (because $\sigma_{11}^* = \sigma_1^*$ and $\sigma_{10}^* > \sigma_1^*$). If he does not work in the first period he will work in the second even if $b_2 = 0$ (because $\sigma_{00}^* < \sigma_1^*$). Hence the payoffs: $(1 + \theta_L - x_{00}^*)(c - \theta_L V)$ is the expected relative loss of the low-type agent from choosing to work in the first period – he loses $(c - \theta_L V)$ in expectation in the first period and after success in the second, that is, with the total probability $1 + \theta_L$; however, he would also lose $(c-\theta_L V)$ with probability x_{00}^* in the second period since he would choose to work even without a bonus. Similarly, the high-type agent gains $(\theta_H V - c)$ with probability $1 + \theta_H$ if he decides to work in the first period but he would also gain $(\theta_H V - c)$ if he chose to shirk since he would work in the second period.

Since the first principal's objective function is the same as that of the principal in the static model, in the case of semi-separation her indifference between offering b^* and offering no bonus to the low-type agent is expressed by (31). In the pooling case she prefers to never offer a bonus: $x_1^* = 1$, and σ_1^* is determined by (32) and satisfies $G_L(\sigma_1^*) < \alpha$.

The proof of the second part of the Lemma is quite similar and is omitted. ■

Lemma 10 *i)In equilibrium there is always semi-separation in the second period after* $b_1 = 0$ *and the agent shirking in the first period; the agent does not always work after* $b_2 = 0$ *in this case:* $x_{00}^* < 1, \sigma_{00}^* < \sigma_1^*$.

ii)In equilibrium there is always pooling after $b_1 = 0$ *and the agent working successfully in the first period; in this case the agent always exerts effort in the second period:* $x_{11}^* = 1, \sigma_{11}^* = \sigma_1^*$.

Proof. i)If there were pooling in this case we would have $\sigma_{00}^* = \sigma_1^*$ from (27) and (30), but $\sigma_{00}^* < \sigma_1^*$ follows from (28) or (29).

ii)From (20) and (30) it follows that the agent will always work ($\sigma_{11}^* = \sigma_1^*$) in this subgame for any policy x_{11}^*. Hence, in equilibrium the principal never gives a bonus in this subgame ($x_{11}^* = 1$). ■

Proof of Proposition 1. In the light of Lemmas 6–9, to prove that (6)–(11) determine an equilibrium (satisfying all required properties), it remains to check that x_1^*, x_{00}^* and x_{10}^* belong to $(0, 1)$, that $\sigma_{00}^* < \sigma_1^* < \sigma_{10}^*$ and that $l(\sigma_1^*) \geq x_1^* \frac{\theta_L}{\theta_H} A$ (so that indeed $\sigma_{11}^* = \sigma_1^*$ and $x_{11}^* = 1$). The last inequality follows immediately from (7).

Note that equations (6), (8) and (10) determine the agent's strategies $\sigma_1^*, \sigma_{00}^*$ and σ_{10}^*, satisfying $\sigma_{00}^* < \sigma_1^* < \sigma_{10}^*$. From the other three equations we find

$$x_1 = \frac{\theta_H l(\sigma_1^*) + l(\sigma_{00}^*)}{A(1+\theta_L)} \tag{37}$$

$$x_{00} = \frac{(1+\theta_L) l(\sigma_{00}^*)}{\theta_H l(\sigma_1^*) + l(\sigma_{00}^*)} \tag{38}$$

$$x_{10} = \frac{1-\theta_H}{1-\theta_L} \frac{(1+\theta_L) l(\sigma_{10}^*)}{\theta_H l(\sigma_1^*) + l(\sigma_{00}^*)} \tag{39}$$

From (38) and $\sigma_{00}^* < \sigma_1^*$ it follows that $x_{00} < \frac{1+\theta_L}{1+\theta_H}$, so that $x_{00} \in (0,1)$. From (39) x_{10} will belong to the unit interval when θ_H is large than some threshold value $\bar{\theta}_H$, which does not depend on f_L. Finally, with other parameters fixed, A grows monotonically and unlimitedly as f_H decreases down to 0. Hence, from (37), $x_1 \in (0,1)$ when f_H is small enough. Here, however, the threshold $\bar{f}$ for f_H depends on θ_H: $\bar{f}(\theta_H)$ is a decreasing function (by the implicit function theorem from (37)).

It remains to prove uniqueness when $(\theta_H, f_H) \in (\bar{\theta}_H, 1) \times (0, \bar{f}(\theta_H))$. By Lemma 10, there is always pooling after success and semi-separation after shirking. Then the only possible candidates are equilibria with pooling after failure and/or pooling in the first period. If there is semi-separation after failure, $f_H < \bar{f}(\theta_H)$ is a necessary and sufficient condition for semi-separation in the first period. Similarly, with semi-separation in the first period $\theta_H > \bar{\theta}_H$ is necessary and sufficient for semi-separation after failure. So we just have to prove that there cannot be an equilibrium with pooling both in the first period and after failure.

If, as assumed, semi-separation occurs in the first period when semi-separation is predicted after failure (since $(\theta_H, f_H) \in (\bar{\theta}_H, 1) \times (0, \bar{f}(\theta_H))$), for pooling to occur in the first period when pooling is predicted after failure we must have

$$2 - \tilde{x}_{00} \geq \frac{1+\theta_L - x_{00}^*}{\theta_H}, \tag{40}$$

where $\tilde{x}_{00}$ is the parameter of the equilibrium with pooling. Since $\tilde{\sigma}_{00}$ is smaller in the pooling equilibrium than σ_{00}^* in the semi-separating (because $\tilde{\sigma}_1$ is smaller than σ_1^*), and $\tilde{x}_1 = 1 > x_1^*$, $\tilde{x}_{00} = l(\tilde{\sigma}_{00})/A < x_{00}^* = l(\sigma_{00}^*)/x_1^* A$, inequality (40) must hold with $\tilde{x}_{00}$ replaced by x_{00}^*. Then, (40) with $\tilde{x}_{00}$ replaced by x_{00}^* is equivalent to

$$l(\sigma_{00}^*) \leq \frac{1+\theta_L - 2\theta_H}{1-\theta_L} l(\sigma_1^*), \tag{41}$$

i.e. θ_H must be small enough. Taking the largest acceptable value $\tilde{\theta}_H$ of θ_H, which makes 41 be satisfied as equality, and subsituting $l(\sigma_{00}^*)$ from 41 (satisfied as equality) into (39) we get $x_{10}^* > 1$, which means $\tilde{\theta}_H < \bar{\theta}_H$ – a contradiction. ■

Proof of Lemma 3. i)The average efforts for the high type in the first and second periods are

$$\bar{e}_1^H = 1 - G_H(\sigma_1^*), \quad (42)$$
$$\bar{e}_2^H = (1 - G_H(\sigma_1^*))\theta_H + (1 - G_H(\sigma_{10}^*))(1 - \theta_H) + (G_H(\sigma_1^*) - G_H(\sigma_{00}^*)). \quad (43)$$

The difference $\bar{e}_2^H - \bar{e}_1^H = (G_H(\sigma_1^*) - G_H(\sigma_{00}^*)) - (1 - \theta_H)(G_H(\sigma_{10}^*) - G_H(\sigma_1^*))$ is positive when θ_H is large enough because $(G_H(\sigma_1^*) - G_H(\sigma_{00}^*)) > 0$ does not depend on θ_H.

For the low type,

$$\bar{e}_1^L = 1 - x_1^* + x_1^*(1 - G_L(\sigma_1^*)), \quad (44)$$
$$\begin{aligned}\bar{e}_2^L = {} & 1 - x_1^* + x_1^*[(1 - G_L(\sigma_1^*))\theta_L + (1 - G_L(\sigma_1^*))(1 - \theta_L)(1 - x_{10}^*) \quad (45)\\ & +(1 - G_L(\sigma_{10}^*))(1 - \theta_L)x_{10}^* + G_L(\sigma_1^*)(1 - x_{00}^*) + (G_L(\sigma_1^*) - G_L(\sigma_{00}^*))x_{00}^*].\end{aligned}$$

The difference

$$\begin{aligned}\bar{e}_2^L - \bar{e}_1^L &= x_1^*[G_L(\sigma_1^*)(1 - x_{00}^*) + (G_L(\sigma_1^*) - G_L(\sigma_{00}^*))x_{00}^*\\ &\quad -(G_L(\sigma_{10}^*) - G_L(\sigma_1^*))(1 - \theta_L)x_{10}^*]\\ &\geq x_1^*[(G_L(\sigma_1^*) - G_L(\sigma_{00}^*)) - (1 - G_L(\sigma_1^*))(1 - \theta_L)x_{10}^*]\end{aligned}$$

will be positive when θ_H is large enough because $x_{10} \to 0$ as $\theta_H \to 1$ and σ_{00}^* and σ_1^* do not depend on θ_H.

ii)Since $\sigma_1^* = \sigma^*$, for the high type $\bar{e}_1^H = \bar{e}^H$. For the low type,

$$\bar{e}_1^L < \bar{e}^L \Leftrightarrow x_1^* > x^* \Leftrightarrow l(\sigma_{00}^*) > (1 + \theta_L - \theta_H)l(\sigma_1^*).$$

The last inequality will be satisfied if θ_H is large enough and θ_L is small enough because from (28) satisfied as equality it follows that σ_{00}^* approaches σ_1^* as θ_L approaches $\frac{c}{V+W}$, its lower bound compatible with Assumption 2. ■

Proof of Lemma 4. i)For the low-type agent

$$\begin{aligned}U_L^P &= U_L^{P1} = \theta_L(W - b^*),\\ U_L^{P2} &= \theta_L(1 - x_1 + x_1[(1 - G_L(\sigma_1^*))(1 - \theta_L) + G_L(\sigma_1^*)])(W - b^*)\\ &\quad +x_1(1 - G_L(\sigma_1^*))\theta_L^2 W) > U_L^{P1}.\end{aligned}$$

ii)For the high type,

$$\begin{aligned}U_H^P &= U_H^{P1} = \theta_H(1 - G_H(\sigma_1^*))W,\\ U_H^{P2} &= [(1 - G_H(\sigma_{00}^*)) - (1 - \theta_H)(G_H(\sigma_{10}^*) - G_H(\sigma_1^*))]\theta_H W.\end{aligned}$$

Since $\sigma_{00}^* < \sigma_1^*$ and $G_H(\sigma_{10}^*) - G_H(\sigma_1^*)$ does not depend on θ_H, for θ_H large enough $U_H^{P2} > U_H^{P1}$. ■

7.2 The model with a permanent principal

Proof of Proposition 2. Assume that in the first period the principal offers no bonus to the high type agent and randomizes between 0 and b^* for the low type. The agent's reaction is the same as in the model with two principals: he works when offered b^* or if self-confident enough when offered 0 (the threshold $\tilde{\sigma}_1$ is determined in the same way as σ_1^* from Proposition 1). Moreover, equations (13), (15) and (17), which determine the agent's reactions are the same as in Proposition 1.

Consider now the principal's objective in the first period. The net benefit of paying no bonus to the weak agent is

$$\begin{aligned}\Delta U_{1,L}^P(\tilde{x}_1) &= (1-G_L(\tilde{\sigma}_1))(\theta_L(W+\theta_L W)+(1-\theta_L)\theta_L(W-b^*)) \\ &\quad +G_L(\tilde{\sigma}_1)\theta_L(W-b^*)-2\theta_L(W-b^*).\end{aligned}$$

This net benefit is equal to 0 when

$$G_L(\tilde{\sigma}_1)=\frac{\alpha(1+\theta_L)}{1+\alpha\theta_L}. \tag{46}$$

Equations (14) and (16) are obtained from (28) and (25) substituting $\tilde{\sigma}_1$ from (46).

To make sure that (12) – (11) determine an equilibrium, we must check that the net benefit of offering no bonus to the high-type agent in the first period

$$\begin{aligned}\Delta U_{1,H}^P(\tilde{x}_1) &= (1-G_H(\tilde{\sigma}_1))(\theta_H(W+\theta_H W)+(1-\theta_H)\theta_H\frac{1-G_H(\tilde{\sigma}_{10})}{1-G_H(\tilde{\sigma}_1)}W \\ &\quad +G_H(\tilde{\sigma}_1)\theta_H\frac{G_H(\tilde{\sigma}_1)-G_H(\tilde{\sigma}_{00})}{G_H(\tilde{\sigma}_1)}W-2\theta_H(W-b^*)\end{aligned}$$

is positive.

Note that from MLRP[33], $\frac{1-G_H(\tilde{\sigma}_{10})}{1-G_H(\tilde{\sigma}_1)}>\frac{1-G_L(\tilde{\sigma}_{10})}{1-G_L(\tilde{\sigma}_1)}$ and $\frac{G_H(\tilde{\sigma}_1)-G_H(\tilde{\sigma}_{00})}{G_H(\tilde{\sigma}_1)}>\frac{G_L(\tilde{\sigma}_1)-G_L(\tilde{\sigma}_{00})}{G_L(\tilde{\sigma}_1)}$. Hence,

$$\begin{aligned}\Delta U_{1,H}^P(\tilde{x}_1) &> (1-G_H(\tilde{\sigma}_1))(\theta_H(W+\theta_H W)+(1-\theta_H)\theta_H(W-b^*)) \\ &\quad +G_H(\tilde{\sigma}_1)\theta_H(W-b^*)-2\theta_H(W-b^*).\end{aligned}$$

Again using MLRP ($G_H(\tilde{\sigma}_1)<G_L(\tilde{\sigma}_1)$) and

$$\frac{\alpha(1+\theta_L)}{1+\alpha\theta_L}<\frac{\alpha(1+\theta_H)}{1+\alpha\theta_H},$$

we obtain $\Delta U_{1,H}^P(\tilde{x}_1)>0$.

[33] The first inequality follows from MLRP and $\tilde{\sigma}_{10}>\tilde{\sigma}_1$ (see note ????????). For the second note that $\frac{a+b}{c+d}<\max\{\frac{a}{c},\frac{b}{d}\}$ for any $a,b,c,d>0$, so $\frac{G_H(\tilde{\sigma}_1)}{G_L(\tilde{\sigma}_1)}=\frac{G_H(\tilde{\sigma}_{00})+(G_H(\tilde{\sigma}_1)-G_H(\tilde{\sigma}_{00}))}{G_L(\tilde{\sigma}_{00})+(G_L(\tilde{\sigma}_1)-G_L(\tilde{\sigma}_{00}))}<\frac{G_H(\tilde{\sigma}_1)-G_H(\tilde{\sigma}_{00})}{G_L(\tilde{\sigma}_1)-G_L(\tilde{\sigma}_{00})}$.

Semi-separation in the first period and after failure will take place if $\theta_H > \tilde{\theta}_H$ and $f_H < \tilde{f}_H(\theta_H)$, where $\tilde{\theta}_H$ and $\tilde{f}_H(\theta_H)$ are defined as in the proof of Proposition 2, but assume values different from $\bar{\theta}_H$ and $\bar{f}_H(\theta_H)$ since thresholds (sigmas) are different now.

It remains to specify out-of equilibrium beliefs, supporting this equilibrium. It suffices to assume that the agent believes he has the low type when offered any bonus different from 0 and b^*. ■

Proof of Proposition 3. (37) determines x_1^* and the same equation with $\tilde{\sigma}_1$ and $\tilde{\sigma}_{00}$ substituted for σ_1^* and σ_{00}^* gives $\tilde{x}_1$. From $\tilde{\sigma}_1 > \sigma_1^*$ and $\tilde{\sigma}_{00} > \sigma_{00}^*$ it follows immediately that $\tilde{x}_1 > x_1^*$. ■

Proof of Proposition 4. i)Average efforts are given by the same formulas as in the model with a permanent principal (**??**)–(45), with σ_i^* replaced by $\tilde{\sigma}_i$ and x_i by $\tilde{x}_i$. The same proof as in Lemma 3 applies.

ii)Comparison of the first-period efforts follows immediately from $\tilde{\sigma}_1 > \sigma_1^*$ and $\tilde{x}_1 > x_1^*$.

For the high type,

$$\begin{aligned}\tilde{e}_2^H - \bar{e}_2^H &= (1-\theta_H)[(G_H(\tilde{\sigma}_1) - G_H(\sigma_1^*)) - (G_H(\tilde{\sigma}_{10}) - G_H(\sigma_{10}^*))] \\ &\quad -(G_H(\tilde{\sigma}_{00}) - G_H(\sigma_{00}^*))\end{aligned}$$

is negative when θ_H is large enough.

For the low type, consider the limit of $\tilde{e}_2^L$ when $\theta_H \to 1:$

$$\lim_{\theta_H \to 1} \tilde{e}_2^L = \lim_{\theta_H \to 1}(1 - \tilde{x}_1\tilde{x}_{10}(G_L(\tilde{\sigma}_{10}) - G_L(\tilde{\sigma}_1)) - \tilde{x}_1\tilde{x}_{00}G_L(\tilde{\sigma}_{00})) = (1 - \frac{l(\tilde{\sigma}_{00})G_L(\tilde{\sigma}_{00})}{A^\circ}),$$

where A° is the value of parameter A at $\theta_H = 1$. With a transient principal

$$\lim_{\theta_H \to 1} \bar{e}_2^L = \lim_{\theta_H \to 1}(1 - x_1^*x_{10}^*(G_L(\sigma_{10}^*) - G_L(\sigma_1^*)) - x_1^*x_{00}^*G_L(\sigma_{00}^*)) = (1 - \frac{l(\sigma_{00}^*)G_L(\sigma_{00}^*)}{A^\circ}).$$

Since $\tilde{\sigma}_{00} > \sigma_{00}^*$, $\lim_{\theta_H \to 1} \tilde{e}_2^L < \lim_{\theta_H \to 1} \bar{e}_2^L$, so for large enough values of θ_H the average effort in the second period will be higher with a transient principal. ■

Proof of Proposition 5. The total welfare of the permanent principal in case of semi-separation is

$$\begin{aligned}\tilde{U}_L^P &= 2\theta_L(W - b^*), \\ \tilde{U}_H^P &= [2(1 - G_H(\tilde{\sigma}_1)) - (1-\theta_H)(G_H(\tilde{\sigma}_{10}) - G_H(\tilde{\sigma}_1)) + (G_H(\tilde{\sigma}_1) - G_H(\tilde{\sigma}_{00}))]\theta_H W.\end{aligned}$$

For the low type,

$$\tilde{U}_L^P - (U_L^{P1} + U_L^{P2}) = -x_1\theta_L^2(1 - G_L(\sigma_1^*))b^* < 0.$$

For the high type,

$$\begin{aligned}\tilde{U}_H^P - (U_H^{P1} + U_H^{P2}) &= [\theta_H(G_H(\sigma_1^*) - G_H(\tilde{\sigma}_1)) + (1-\theta_H)(G_H(\sigma_{10}^*) - G_H(\tilde{\sigma}_{10})) \\ &\quad + (G_H(\sigma_{00}^*) - G_H(\tilde{\sigma}_{00}))]\theta_H W < 0\end{aligned}$$

because $\tilde{\sigma}_i > \sigma_i^*, i \in \{1, 00, 10\}$. ■

Proof of Lemma 5. The two-period welfare of the high-type agent facing transient principals is

$$\begin{aligned}U_H^{A1} + U_H^{A2} &= [2(1 - G_H(\sigma_1^*)) + (G_H(\sigma_1^*) - G_H(\sigma_{00}^*)) \\ &\quad -(1-\theta_H)(G_H(\sigma_{10}^*) - G_H(\sigma_1^*))](\theta_H V - c).\end{aligned}$$

With a permanent principal the agent's welfare is given by the same formula with appropriate signal thresholds $\tilde{\sigma}_1, \tilde{\sigma}_{00}$ and $\tilde{\sigma}_{10}$. Then,

$$\begin{aligned}(U_H^{A1} + U_H^{A2}) - (\tilde{U}_H^{A1} + \tilde{U}_H^{A2}) &= [(G_H(\tilde{\sigma}_1) - G_H(\sigma_1^*))\theta_H + (G_H(\tilde{\sigma}_{10}) - G_H(\sigma_{10}^*))(1-\theta_H) \\ &\quad + (G_H(\tilde{\sigma}_{00}) - G_H(\sigma_{00}^*))](\theta_H V - c)\end{aligned}$$

is positive. ■

7.3 Alternative specifications

7.3.1 Unknown valuation learned after success

Denote $\hat{\sigma}_1, \hat{\sigma}_{00}$ and $\hat{\sigma}_{10}$ the solutions of the following equations:

$$G_L(\sigma_1) = \alpha, \tag{47}$$

$$G_L(\sigma_{00}) = \alpha^2, \tag{48}$$

$$G_L(\sigma_{10}) = \alpha(2-\alpha) \tag{49}$$

and

$$\begin{aligned}\bar{\theta} &= \frac{l(\hat{\sigma}_{10}) - l(\hat{\sigma}_{00})}{l(\hat{\sigma}_1)}, \\ \underline{\theta} &= \frac{l(\hat{\sigma}_1) - l(\hat{\sigma}_{00})}{l(\hat{\sigma}_1)}; \\ A &= \left(\frac{f_L}{f_H}\right)\left(\frac{c - \theta V_L}{\theta V_H - c}\right).\end{aligned}$$

Lemma 11 *Assume that f_L is large enough so that there is partial separation in the first period.*

i)If $\bar{\theta} < 1$ and $\theta \in (\bar{\theta}, 1]$, in the equilibrium there is partial separation after a failure in the first period ($x_{10} < 1$) and the class of fragile agents is non-empty

($\sigma_1 < \sigma_{10}$). The equilibrium signal thresholds are $\hat{\sigma}_1, \hat{\sigma}_{00}$ and $\hat{\sigma}_{10}$ and the principal's policy parameters x_1, x_{00} and x_{10} are determined implicitly by

$$\begin{aligned} l(\sigma_1) &= x_1 \frac{1 - x_{00}}{\theta} A, \\ l(\sigma_{00}) &= x_1 x_{00} A, \\ l(\sigma_{10}) &= x_1 x_{10} A. \end{aligned}$$

ii)If $\theta \in (\underline{\theta}, \min\{\bar{\theta}, 1\}]$, in the equilibrium there is pooling after a failure in the first period ($x_{10} = 1$: no bonus is offered to both types) but the class of fragile agents is non-empty ($\sigma_1 < \sigma_{10}$). The equilibrium signal thresholds are $\hat{\sigma}_1, \hat{\sigma}_{00}$ and $\sigma_{10} = x_1 A$ and the principal's policy parameters x_1 and x_{00} are determined implicitly by

$$\begin{aligned} l(\sigma_1) &= x_1 \frac{1 - x_{00}}{\theta} A, \\ l(\sigma_{00}) &= x_1 x_{00} A. \end{aligned}$$

iii)If $\theta \in (0, \underline{\theta}]$, in the equilibrium there is pooling after a failure in the first period ($x_{10} = 1$: no bonus is offered to both types) and the class of fragile agents is empty ($\sigma_1 = \sigma_{10}$). The equilibrium signal thresholds are $\hat{\sigma}_1, \hat{\sigma}_{00}$ and the principal's policy parameters x_1 and x_{00} are determined implicitly by

$$\begin{aligned} l(\sigma_1) &= x_1 (2 - x_{00} - \theta) A, \\ l(\sigma_{00}) &= x_1 x_{00} A. \end{aligned}$$

The proof of Lemma 11 is similar to the analysis in Section 4 and is omitted.

7.3.2 Unknown valuation learned after the second period

Lemma 12 *Assume that f_L is large enough.*

i)In the model with a transient principal, there exists a partially separating equilibrium with parameters $\sigma_1^*, \sigma_{00}^*, x_1$ and x_{00} determined from (47), (48) and

$$l(\sigma_1) = x_1 (2 - x_{00}) A, \tag{50}$$
$$l(\sigma_{00}) = x_1 x_{00} A. \tag{51}$$

ii)With a long-lasting principal, there exists a partially separating equilibrium with parameters $\tilde{\sigma}_1, \tilde{\sigma}_{00}, \tilde{x}_1$ and $\tilde{x}_{00}$ determined from (48), (50), (51) and

$$G_L(\sigma_1) = \frac{2\alpha}{1 + \alpha}. \tag{52}$$

7.3.3 Lump-sum transfers

Lemma 13 *If* $(\tilde{a}, \tilde{b})$ *is a contract offered in equilibrium to the high-type agent with a positive probability, then* $\tilde{b} = 0$.

Proof. Assume that $\tilde{b} > 0$ and the principal is indifferent between offering $(\tilde{a}, \tilde{b})$ or deviating to $(\tilde{a}, 0)$ when the agent has the low type. That is, when offered $(\tilde{a}, 0)$ the agent works if $\sigma \geq \hat{\sigma}$, where

$$\theta_L(1 - G_L(\tilde{\sigma}))(W - \tilde{b}) - \tilde{a} = \theta_L(1 - G_L(\hat{\sigma}))W - \tilde{a}, \tag{53}$$

i.e.

$$\frac{1 - G_L(\tilde{\sigma})}{1 - G_L(\hat{\sigma})} = \frac{W}{W - \tilde{b}}. \tag{54}$$

But since $\hat{\sigma} \geq \tilde{\sigma}$ from (53), MLRP implies that

$$\frac{1 - G_H(\tilde{\sigma})}{1 - G_H(\hat{\sigma})} < \frac{1 - G_L(\tilde{\sigma})}{1 - G_L(\hat{\sigma})} \tag{55}$$

so the principal should prefer to deviate to $(\tilde{a}, 0)$ when $\theta = \theta_H$ given the agent's reaction $\hat{\sigma}$. The NWBR criterion then implies that the agent should believe that he is the high type when offered $(\tilde{a}, 0)$, which, in turn, makes the principal indeed willing to deviate to $(\tilde{a}, 0)$ when the agent has the high type – a contradiction, so $\tilde{b} = 0$. ∎

Lemma 14 *Assume that a contract* $(\tilde{a}, \tilde{b})$ *is offered in equilibrium to each type of agent with positive probability, and let* $\tilde{\sigma}$ *be the agent's reaction to this offer. Then* $\tilde{a} \leq a^* = c - \theta_L V$, $\tilde{b} = 0$ *and at least one of the following conditions is satisfied:*
i) $\tilde{\sigma} = 0$ *and* $l(0) \geq \frac{\theta_L}{\theta_H}$;
ii)$\tilde{a} = 0$ *and* $l(\tilde{\sigma}) \leq \frac{\theta_L}{\theta_H}$;
iii)$l(\tilde{\sigma}) = \frac{\theta_L}{\theta_H}$.

Proof. Bonus $\tilde{b} = 0$ by Lemma 13.To see that $\tilde{a} \leq a^*$ note that when the agent has the low type, the principal prefers a contract $(0, b^*)$ to any contract (a, b) with $a > a^*$ (because when $\theta = \theta_L$ she is indifferent between $(0, b^*)$ and $(a^*, 0)$ even when the agent works with probability 1 after receiving the latter offer).

Assume that $\tilde{\sigma} = 0$. If $\tilde{a} = 0$, at least one of the statements i) or ii) is true. So suppose that $\tilde{\sigma} = 0$ and $\tilde{a} > 0$. The principal should not want to deviate to some transfer $\tilde{a} - \varepsilon$ with $\varepsilon > 0$. Assume that $\hat{\sigma}(\varepsilon)$ is the agent's reaction to $(\tilde{a} - \varepsilon, 0)$ which makes the principal indifferent between deviating or not when $\theta = \theta_L$:

$$\theta_L W - \tilde{a} = \theta_L(1 - G_L(\hat{\sigma}))W - \tilde{a} + \varepsilon. \tag{56}$$

The principal should not want to deviate when $\theta = \theta_H$ if the agent's reaction is $\hat{\sigma}(\varepsilon)$, otherwise, according to NWBR, the agent would believe that he is the high type when offered $(\tilde{a} - \varepsilon, 0)$ and $(\tilde{a}, 0)$ would not be an equilibrium bonus. Thus

$$\theta_H W - \tilde{a} \geq \theta_H(1 - G_H(\hat{\sigma}))W - \tilde{a} - \varepsilon. \tag{57}$$

Subtracting (56) from (57), we get

$$\frac{G_H(\hat{\sigma}(\varepsilon))}{G_L(\hat{\sigma}(\varepsilon))} > \frac{\theta_L}{\theta_H}. \tag{58}$$

From (56) it follows that $\hat{\sigma}(\varepsilon) \to 0$ as $\varepsilon \to 0$, so taking the limit $\varepsilon \to 0$ in (58) we obtain

$$l(0) \geq \frac{\theta_L}{\theta_H}. \tag{59}$$

Assume now that $\tilde{\sigma} > 0$ and $\tilde{a} = 0$. It must be the case that the principal does not want to deviate to a wage $\tilde{a} + \varepsilon = \varepsilon$, that is such a deviation should not be interpreted by the agent as a signal $\theta = \theta_H$ for all ε small enough. If the principal is indifferent between $(0, 0)$ and $(\varepsilon, 0)$ when $\theta = \theta_L$,

$$\theta_L(1 - G_L(\tilde{\sigma}))W = \theta_L(1 - G_L(\hat{\sigma}))W - \varepsilon, \tag{60}$$

it must be that

$$\theta_H(1 - G_H(\tilde{\sigma}))W \geq \theta_H(1 - G_H(\hat{\sigma}))W - \varepsilon. \tag{61}$$

Subtracting (60) from (61) and taking the limit $\varepsilon \to 0$, we get

$$l(\tilde{\sigma}) \leq \frac{\theta_L}{\theta_H}. \tag{62}$$

Finally, when $\tilde{\sigma} > 0$ and $\tilde{a} > 0$ it must be the case that the principal wants to deviate to none of $(\tilde{a} - \varepsilon, 0)$ and $(\tilde{a} + \varepsilon, 0)$. Reasoning similar to the previous cases shows that

$$l(\tilde{\sigma}) = \frac{\theta_L}{\theta_H}.$$

■

Lemma 15 *In any equilibrium (satisfying NWBR) a single contract is offered to the high type agent.*

Proof. Assume that $(\tilde{a}_1, 0)$ and $(\tilde{a}_2, 0)$ are two different contracts which are offered to the high type agent, with $\tilde{a}_1 < \tilde{a}_2$. Let $\tilde{\sigma}_1$ and $\tilde{\sigma}_2$ be the agent's reactions to these bonuses (necessarily satisfying $\tilde{\sigma}_1 > \tilde{\sigma}_2$). At least one of the contracts $\tilde{a}_1$

or $\tilde{a}_2$ must be offered with a positive probability to the low type agent as well. If both are, then

$$\theta_L(1 - G_L(\tilde{\sigma}_1))W - \tilde{a}_1 = \theta_L(1 - G_L(\tilde{\sigma}_2))W - \tilde{a}_2, \tag{63}$$

$$\theta_H(1 - G_H(\tilde{\sigma}_1))W - \tilde{a}_1 = \theta_H(1 - G_H(\tilde{\sigma}_2))W - \tilde{a}_2. \tag{64}$$

Subtracting (63) from (64), we get

$$\frac{G_H(\tilde{\sigma}_1) - G_H(\tilde{\sigma}_2)}{G_L(\tilde{\sigma}_1) - G_L(\tilde{\sigma}_2)} = \frac{\theta_L}{\theta_H}.$$

From MLRP it follows that

$$l(\tilde{\sigma}_2) < \frac{G_H(\tilde{\sigma}_1) - G_H(\tilde{\sigma}_2)}{G_L(\tilde{\sigma}_1) - G_L(\tilde{\sigma}_2)} < l(\tilde{\sigma}_1),$$

so $l(\tilde{\sigma}_2) < \frac{\theta_L}{\theta_H}$ and $l(\tilde{\sigma}_1) > \frac{\theta_L}{\theta_H}$, which makes the principal willing to deviate to $\tilde{a}_1 + \varepsilon$ and $\tilde{a}_2 - \varepsilon$ – a contradiction.

Of only $\tilde{a}_2$ is offered to the low type, we get

$$\frac{G_H(\tilde{\sigma}_1) - G_H(\tilde{\sigma}_2)}{G_L(\tilde{\sigma}_1) - G_L(\tilde{\sigma}_2)} < \frac{\theta_L}{\theta_H},$$

so $l(\tilde{\sigma}_2) < \frac{\theta_L}{\theta_H}$ and the principal would wish to deviate to $\tilde{a}_2 - \varepsilon$ from $\tilde{a}_2 - \varepsilon$ (remember $\tilde{a}_2 > \tilde{a}_1 \geq 0$).

Finally, if only $\tilde{a}_1$ is offered when $\theta = \theta_L$, we obtain similarly $l(\tilde{\sigma}_1) > \frac{\theta_L}{\theta_H}$, which implies $\tilde{\sigma}_1 = 0$ by Lemma 14, contradicting to $\tilde{\sigma}_1 > \tilde{\sigma}_2$. To complete the proof just note that there cannot be two different wages offered to the high type agent only, so we have analyzed all possible cases. ■

Corollary 5 *There are three possible types of equilibrium:*

i)pooling: the principal always offers $(\tilde{a}, 0)$ for some $\tilde{a} \in [0, a^*]$;

ii)semi-separating: the principal always offers $(\tilde{a}, 0)$ for some $\tilde{a} \in [0, a^*]$ to the high type and with probability $x \in (0, 1)$ to the low type; $(0, b^*)$ is offered to the low type with probability $1 - x$;

iii)separating: the principal always offers $(a^*, 0)$ when $\theta = \theta_H$ and $(0, b^*)$ when $\theta = \theta_L$.

Proof. The only statement in the corollary that does not follow directly from the previous analysis is that the only contract that can be offered to the high type agent in a separating equilibrium is $(a^*, 0)$. But the wage $\tilde{a}$ cannot be lower than a^* – otherwise the principal would also offer it to the low type. Neither can $\tilde{a}$ be higher – the principal could offer a wage between a^* and $\tilde{a}$, which would be still interpreted as a signal that $\theta = \theta_H$. ■

Proof of Proposition 7. Assume that $l_0 > A(f_H)$ and the principal always offers a pooling contract $(0,0)$. The best response of the agent is to always work. This is clearly an equilibrium since the principal cannot gain by any deviation. There can be no other pooling or semi-separating equilibrium with an offer $(a,0)$ with $a > 0$: the agent would always work in response and the principal would be able to deviate profitably to $a - \varepsilon$ because $l_0 < \frac{\theta_L}{\theta_H}$ (see Lemma 14). Separating equilibrium is impossible for the same reason: the principal would wish to deviate to $a^* - \varepsilon$ when $\theta = \theta_H$.

Now we shall consider the case $l_0 < A(f_H)$. Let us suppose that $W > \bar{W}$ and $f_H > \tilde{f}_H(W)$ (i.e. $(l_0, f_H) \in \mathcal{R}_{2A}$) and prove that a pooling contract $(0,0)$ with the agent's best response $\tilde{\sigma}$, determined by $l(\tilde{\sigma})$ form an equilibrium. Suppose that the principal deviates to $(\hat{a},\hat{b})$ and the agent's response $\hat{\sigma}$ to $(\hat{a},\hat{b})$ makes the principal indifferent between deviating or not when $\theta = \theta_L$:

$$\theta_L(1 - G_L(\tilde{\sigma}))W = \theta_L(1 - G_L(\hat{\sigma}))(W - \hat{b}) - \hat{a}. \tag{65}$$

It is sufficient to check that the principal strictly prefers not to deviate when $\theta = \theta_H$,

$$\theta_H(1 - G_H(\tilde{\sigma}))W > \theta_H(1 - G_H(\hat{\sigma}))(W - \hat{b}) - \hat{a}, \tag{66}$$

for, then the agent will infer that $\theta = \theta_L$ from an offer of $(\hat{a},\hat{b})$ and the principal will not wish to deviate. Subtracting (65) from (66), we see that (66) is equivalent to

$$\begin{aligned} & (\theta_H(G_H(\tilde{\sigma}) - G_H(\hat{\sigma})) - \theta_L(G_L(\tilde{\sigma}) - G_L(\hat{\sigma})))W \\ < \; & (\theta_H(1 - G_H(\hat{\sigma})) - \theta_L(1 - G_L(\hat{\sigma})))\hat{b}. \end{aligned} \tag{67}$$

The LHS of (67) is negative because

$$\frac{G_H(\tilde{\sigma}) - G_H(\hat{\sigma})}{G_L(\tilde{\sigma}) - G_L(\hat{\sigma})} < l(\tilde{\sigma}) < \frac{\theta_L}{\theta_H} \tag{68}$$

(note that $\tilde{\sigma} > \hat{\sigma}$ from (65)). The first inequality in (68) follows from MLRP, the second from $f_H > \tilde{f}_H(W) > \bar{f}_H$, so that $l(\tilde{\sigma}) = A(f_H) < \frac{\theta_L}{\theta_H}$. The RHS of (67) is positive because $\theta_H > \theta_L$ and $1 - G_H(\hat{\sigma}) > 1 - G_L(\hat{\sigma})$ due to MLRP.

Thus, the only deviation that could be profitable is to $(0, b^*)$, which would induce the agent's working with certainty. The principal does not gain from such a deviation,

$$\theta_L(W - b^*) < \theta_L(1 - G_L(\tilde{\sigma})),$$

because firstly, from $f_H > \bar{f}_H$ it follows that $\tilde{\sigma} < \hat{\sigma}$, where $l(\hat{\sigma}) = \frac{\theta_L}{\theta_H}$, and secondly, $G_L(\hat{\sigma}) < \frac{b^*}{W}$ because $f_H > \tilde{f}_H(W)$.

Let us now prove that there are no other equilibria in this case. Indeed, an offer $(\hat{a}, 0)$ with $\hat{a} > 0$ cannot a part of a pooling or semi-separating equilibrium because the agent's reaction $\hat{\sigma}$ will be determined by $l(\hat{\sigma}) = x_{A(f_H)}$ for some $x \in [0,1]$ and thus would satisfy $l(\hat{\sigma}) < A(f_H) < \frac{\theta_L}{\theta_H}$, which contradicts Lemma 14. In the separating equilibrium l_0 must not be smaller than $\frac{\theta_L}{\theta_H}$ – otherwise the principal would offer $a^* - \varepsilon$ when $\theta = \theta_H$ (by reasoning similar to the proof of Lemma 14), so there cannot be a separating equilibrium in our case.

We omit analysis of another three cases, $\mathcal{R}_{3A}$, $\mathcal{R}_{2B}$ and $\mathcal{R}_{3B}, \mathcal{R}_4$ and $\mathcal{R}_5$ since it is very similar to the one just performed and as lengthy. ■

Proof of Proposition 8. Let us assume that $l_0 = 0$ for the simplicity of referring to the results concerning the model without lump-sum transfers; it will be clear that the proposition still holds for positive l_0 small enough. It can be proved that after any contract offered in the first period the agent will work if and only if his signal is above some threshold. This implies that each subgame in the second period has the same structure as the static game with parameters appropriately modified. In particular, the high type agent is never offered a bonus in the second period and is always offered a single contract in equilibrium.

Assume that a contract (a_1, b_1) is offered in the first period with probability $x_1^L > 0$ to the low type agent and with probability $x_1^H > 0$ to the high type, and let σ_1^* be the agent's reaction.

Consider a subgame which is played after the agent was offered (a_1, b_1), exerted effort and succeeded in the first period. The principal believes that a signal of the agent of type i has a cdf $\frac{G_i(\sigma) - G_i(\sigma_1^*)}{1 - G_i(\sigma_1^*)}$ on $[\sigma_1^*, \infty)$; then, the likelihood ratio of this distribution, $\tilde{l}^{11}(\sigma) = l(\sigma)\frac{1-G_L(\sigma_1^*)}{1-G_H(\sigma_1^*)}$. Assume that a contract $(a_{11}, 0)$ is always offered to the high type agent and with probability $x_{11} \in [0,1]$ to the low type. The appropriate parameter, defining the agent's reaction in this subgame is $\tilde{A}^{11} = \frac{x_1^L}{x_1^H}\frac{\theta_L}{\theta_H}\frac{1-G_H(\sigma_1^*)}{1-G_L(\sigma_1^*)}A$, so that the agent works whenever $\tilde{l}^{11}(\sigma) \geq x_{11}\tilde{A}^{11}$ (the term $\frac{\theta_L}{\theta_H}$ comes from the agent's updating of his beliefs after success, and the term $\frac{1-G_H(\sigma_1^*)}{1-G_L(\sigma_1^*)}$ appears because we replaced $l(\sigma)$ by $\tilde{l}^{11}(\sigma)$). We shall later check that under assumptions made in the proposition the pair $(\tilde{l}^{11}(\sigma_1^*), f_H)$ belongs to $\mathcal{R}_1$ from Proposition 7 so that there is a pooling equilibrium with contract $(0,0)$ and the agent always working.

Now let us look at the subgame which is played after a failure. Now the likelihood ratio is $\tilde{l}^{10}(\sigma) = l(\sigma)\frac{1-G_L(\sigma_1^*)}{1-G_H(\sigma_1^*)}$ and the agent's reaction parameter $\tilde{A}^{10} = \frac{x_1^L}{x_1^H}\frac{1-\theta_L}{1-\theta_H}\frac{1-G_H(\sigma_1^*)}{1-G_L(\sigma_1^*)}A$. Again, we shall later check that the pair $(\tilde{l}^{10}(\sigma_1^*), f_H)$ belongs to $\mathcal{R}_{3A}$ from Proposition 7 so that there is a semi-separating equilibrium with contract $(0,0)$ and the agent working whenever $\sigma \geq \sigma_{10}^*$, such that $\frac{G_L(\sigma_{10}^*) - G_L(\sigma_1^*)}{1-G_L(\sigma_1^*)} = \frac{b^*}{W}$.

Finally, in the subgame arising after the agent's shirking in the first period the principal believes that a signal of the agent of type i has a cdf $\frac{G_i(\sigma)}{G_i(\sigma_1^*)}$ on $[0, \sigma_1^*)$; then, the likelihood ratio of this distribution is $\tilde{l}^{00}(\sigma) = l(\sigma)\frac{G_L(\sigma_1^*)}{G_H(\sigma_1^*)}$. The agent's reaction parameter $\tilde{A}^{00} = \frac{x_1^L}{x_1^H}\frac{G_H(\sigma_1^*)}{G_L(\sigma_1^*)}A$. Again, we shall check that $(\tilde{l}^{10}(\sigma_1^*), f_H)$ belongs to $\mathcal{R}_{3A}$ from Proposition 7 so that there is a semi-separating equilibrium with contract

$(0,0)$ and the agent working whenever $\sigma \geq \sigma^*_{00}$, such that $\frac{G_L(\sigma^*_{00})}{G_L(\sigma^*_1)} = \frac{b^*}{W}$.

Let us now turn to the analysis of the first-period interaction. It looks very similar to the one described by the static model, the principal objective, in particular, being the same. The difference is in the parameter $\tilde{A}^1$, defining the agent's reaction in the first period. Now $\tilde{A}^1$ is determined endogenously since the relative attractiveness of working versus shirking depends upon the second principal's policy, which, in turn, depends on the first principal's one. In particular, when the equilibria in the second-period subgames are as described above, $\tilde{A}^1 = \frac{1+\theta_L - x_{00}}{\theta_H} A$ (see Lemma 9). Since x_{00} determined in Proposition 1 for the model without lump-sum transfers (for f_H small enough and θ_H large enough) does not depend on f_H (see the proof of Proposition 1), there exists $\bar{f}^1_H$ such that $\tilde{A}^1(\bar{f}^1_H) = \frac{\theta_L}{\theta_H}$. Then $(l_0, f_H) \in \mathcal{R}_{3A}$ (in notation of Proposition 7) in the first-period game whenever $f_H < \bar{f}^1_H$. It remains to take W large enough so that, firstly, $W > \bar{W}$ and, secondly, $\tilde{l}^{11}(\sigma^*_1) = \tilde{l}^{10}(\sigma^*_1) < \frac{\theta_L}{\theta_H}$ (this is possible since $\tilde{l}^{10}(\sigma^*_1) \to 0$ as $\sigma^*_1 \to 0$, and $\sigma^*_1 \to 0$ when $W \to \infty$). Since $\frac{1+\theta_L - x_{00}}{\theta_H} < x_1 \frac{1-\theta_L}{1-\theta_H}$ for θ_H large enough, $(\tilde{l}^{10}(\sigma^*_1), f_H)$ belongs to $\mathcal{R}_{3A}$. Because $\frac{1+\theta_L - x_{00}}{\theta_H} > x_1 \frac{\theta_L}{\theta_H}$, $(\tilde{l}^{11}(\sigma^*_1), f_H)$ belongs to $\mathcal{R}_1$. ■

8 Appendix B

Example 1. This example shows that the low-type agent can have a higher and a lower payoff in a long-term relationship. For simplicity assume that $\theta_H = 1$ so that $\sigma^*_{10} = \tilde{\sigma}_{10} = \infty$ and $x_{10} = \tilde{x}_{10} = 0$. Then, the low-type agent's expected utility with transient principals and with a permanent principal is respectively

$$\begin{aligned} U^{A1}_L + U^{A2}_L &= -[x_1(1 - G_L(\sigma^*_1))(1 + \theta_L - x_{00}) + x_1 x_{00}(1 - G_L(\sigma^*_{00}))](c - \theta_L V); \\ \tilde{U}^{A1}_L + \tilde{U}^{A2}_L &= -[\tilde{x}_1(1 - G_L(\tilde{\sigma}_1))(1 + \theta_L - \tilde{x}_{00}) + \tilde{x}_1 \tilde{x}_{00}(1 - G_L(\tilde{\sigma}_{00}))](c - \theta_L V). \end{aligned}$$

Substituting (37)-(39) and the corresponding formulas for $\tilde{x}_i$, as well as equilibrium levels of σ^*_i and $\tilde{\sigma}_i$ from Propositions 2 and 1, with simple manipulations we obtain that

$$U^{A1}_L + U^{A2}_L > \tilde{U}^{A1}_L + \tilde{U}^{A2}_L$$

if and only if

$$(1 + \alpha\theta_L)(l(\sigma^*_1) + (1+\alpha) l(\sigma^*_{00})) < (l(\tilde{\sigma}_1) + (1 + \alpha + \alpha\theta_L) l(\tilde{\sigma}_{00})). \tag{69}$$

Take $G_H(\sigma) = \sigma^2$ and $G_L(\sigma) = \sigma(2-\sigma)$ and $\sigma \in (0,1)$. Fix $\theta_L = 0.5, V = 1$; then one can show that inequality (69) will be satisfied if W is large enough (that is α small enough). For example, a relationship with two transitory principals is better for the low-type agent if $\alpha = 0.5$ and worse if $\alpha = 0.9$.

Example 2. This example shows that in the setting of Section 5.2 a long-term principal may offer in the first period a higher bonus to the high-type agent than to the low-type one. More precisely, in the equilibrium with the first-period bonus belonging to $\{0, b^*\}$, the low-type agent will never receive a bonus in the first period while the high type may be offered b^* or no bonus. To simplify things, let us take $\theta = 1$ so that the agent always succeeds in the task whenever he tries to accomplish it. Then, if the agent exerts effort in the first period, any continuation game is virtually the one of complete information (the principal's ignorance about the agent's self-confidence becomes irrelevant), so in the second period the agent receives bonus b^* if $V = V_L$ and no bonus if $V = V_H$ and always works. If the agent was idle in the first period, in the second the principal as usual will randomize between b^* and no bonus when $V = V_L$ and never give a bonus when $V = V_H$. The second-period agent's strategy (when he was idle in the first and received no bonus offer in the second) as a function of the first-period one, $\sigma_{00}^*(\sigma_1^*)$, is defined implicitly by $G_L(\sigma_{00}^*) = \alpha G_L(\sigma_1^*)$ (see the proof of Lemma 8). The net principal's benefit of paying no bonus rather than b^* in the first period is

$$\begin{aligned}\Delta U_{1,L}^P(\sigma_1^*) &= (1 - G_L(\sigma_1^*))(W + W - b^*) + G_L(\sigma_1^*)(W - b^*) - 2(W - b^*) \\ &= W[(1 - G_L(\sigma_1^*)) - (1 - \alpha)]\end{aligned}$$

when the agent has the low type and

$$\begin{aligned}\Delta U_{1,H}^P(\sigma_1^*) &= 2(1 - G_H(\sigma_1^*))W + (G_H(\sigma_1^*) - G_H(\sigma_{00}^*(\sigma_1^*)))W - (2W - b^*) \\ &= W[(1 - G_H(\sigma_1^*)) + (1 - G_H(\sigma_{00}^*(\sigma_1^*))) - (2 - \alpha)]\end{aligned}$$

when the agent has the high type.

Consider the agent's first-period strategy σ_1^* that makes the principal indifferent between paying b^* and giving no bonus when $V = V_H$, that is $\Delta U_{1,H}^P(\sigma_1^*) = 0$, or equivalently $G_H(\sigma_1^*) + G_H(\sigma_{00}^*(\sigma_1^*)) = \alpha$. Note that $\sigma_{00}^*(\sigma_1^*)$ approaches σ_1^* as α approaches 1. Hence, $G_H(\sigma_1^*) \to \frac{1}{2}$ as $\alpha \to 1$.

When $V = V_L$, the principal prefers to offer no bonus if $\Delta U_{1,L}^P(\sigma_1^*)$ is positive, i.e. $G_L(\sigma_1^*) < G_H(\sigma_1^*) + G_H(\sigma_{00}^*(\sigma_1^*))$. The last inequality will be satisfied for α close enough to 1 if there exists $\varepsilon > 0$ such that $G_H(\sigma) > (\frac{1}{2} + \varepsilon)G_L(\sigma)$ for all $\sigma \in (\bar{\sigma} - \varepsilon, \bar{\sigma} + \varepsilon)$, where $\bar{\sigma}$ is the solution of $G_H(\sigma) = \frac{1}{2}$. This condition will be satisfied if, for example, $G_H(\sigma) = \sigma^2$ and $G_L(\sigma) = \sigma(2 - \sigma)$ and $\sigma \in (0, 1)$.

It remains to check that the principal indeed prefers to mix no bonus and b^* when $V = V_H$. The principal's strategies x_{00} (the probability of offering in the second period no bonus to the idle agent when $V = V_L$) and x_1^H (the probability of offering no bonus in the first period when $V = V_H$) satisfy

$$\begin{aligned}l(\sigma_{00}^*) &= \frac{x_{00}}{x_1^H}A, \\ l(\sigma_1^*) &= \frac{1}{x_1^H}(1 - x_{00})A.\end{aligned}$$

For x_1^H to belong to the unit interval, it suffices that A be small enough, that is f_H, the probability of V_H, be large enough.

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